

Introduction To Algebra

~The Real Numbers~

↓ -7 blank 5

- Solution -

Negative numbers are smaller than positive numbers.

$$-7 < 5 \checkmark$$

2, Simplify the expression and combine like terms.

$$-3(3y - 5) + 2(2y + 5)$$

- Solution -

$$-3(3y - 5) + 2(2y + 5)$$

Distribute first:

$$-9y + 15 + 4y + 10$$

Combine like terms:

$$-9y + 4y + 15 + 10$$

$$= -5y + 25 \checkmark$$

3, Name the rational numbers from the list below:

$$-10, 0.98, 4.65, \sqrt{8}, \sqrt{4}, 4\frac{1}{3}, -\frac{15}{49}$$

$$1.27227727772$$

- Solution -

Rational numbers are all the top numbers except $\sqrt{8}$ and 1.27227727772 .
 $\sqrt{4}$ is rational because it is = 2.

4, Simplify: $9 - (-4) - 10 + 6$

- Solution -

$$9 - (-4) - 10 + 6$$

work from left to right.

$$\begin{aligned} & 9 + 4 - 10 + 6 \\ & = 13 - 10 + 6 = 3 + 6 = 9 \checkmark \end{aligned}$$

5, find the value of: $| -13 | - | -5 |$.

- Solution -

$$| -13 | = 13$$

$$| -5 | = 5$$

$$\text{Therefore } 13 - 5 = 8 \checkmark$$

6) J. D. Partin enjoys playing Triominoes every Wednesday night. His scores were:

$$-7, 8, -13 \text{ and } 19.$$

What is his final score?

- Solution -

Add the whole results:

$$-7 + 8 + -13 + 19$$

$$= 1 + 13 + 19$$

$$= -12 + 19 = 7 \checkmark$$

7) Evaluate the expressions:

$$mp + p^3, m = \frac{-5}{3}, p = \frac{1}{3}.$$

- Solution -

Replace m with $\frac{-5}{3}$ and p with $\frac{1}{3}$.

$$\left(\frac{-5}{3}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^3$$

$$= \frac{-5}{9} + \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{-5}{9} + \frac{1}{27}$$

$$\text{Rewrite } \frac{-5}{9} \text{ as } \frac{3}{3} \times \frac{-5}{9} = \frac{-15}{27}.$$

$$\frac{-15}{27} + \frac{1}{27} = \frac{-14}{27} \checkmark$$

8) Simplify: $-3(5)^2 - (-4)(-7)$

- Solution -

$$-3(5)^2 - (-4)(-7)$$

- Use order of operations.
Parenthesis should be done first:

$$-3(25) - (-4)(-7)$$

Do multiplication next:

$$-75 - 28 = -103 \checkmark$$

9) Perform the indicated operations.

$$\frac{6(2^3 - 3) - 5(-3^3 + 14)}{4[2 - (-3)]}$$

- Solution -

Do parenthesis first:

$$(2^3 - 3) = 2 \times 2 \times 2 - 3 = 5 \checkmark$$

$$(-3^3 + 14) = -27 + 14 = -13 \checkmark$$

$$[2 - (-3)] = 2 + 3 = 5$$

Replace those values back in the problem:

$$\frac{6(5) - 5(-13)}{4[5]} = \frac{30 + 65}{20} = \frac{95}{20}$$

Divide both sides by 5.

$$\frac{19}{4} \checkmark$$

10, Convert the following phrase into a mathematical expression.

A number multiplied by -8 , Subtracted from the sum of 9 and 6 times the number.

- Solution -

$$(9 + 6x) - (-8x) \checkmark$$

11, Determine whether 9 is a solution of the equation.

$$2z + 7(z - 3) = 60$$

- Solution -

Replace z with 9 in the equation.

$$2(9) + 7(9 - 3) \stackrel{?}{=} 60$$

$$18 + 7(6) \stackrel{?}{=} 60$$

$$18 + 42 \stackrel{?}{=} 60$$

$$60 = 60 \checkmark$$

Therefore 9 is a solution to the equation.

12, Add the following: $5\frac{3}{4} + (-6\frac{1}{2})$

- Solution -

$$5\frac{3}{4} - 6\frac{1}{2}$$

Convert to improper fractions:

$$5\frac{3}{4} \text{ becomes } \frac{5 \times 4 + 3}{4} = \frac{23}{4}$$

$$6\frac{1}{2} \text{ becomes } \frac{6 \times 2 + 1}{2} = \frac{13}{2}$$

$$\frac{23}{4} - \frac{13}{2} . \quad \text{Rewrite } \frac{13}{2} \text{ as } \frac{2}{2} \times \frac{13}{2} = \frac{26}{4}.$$

$$\frac{23}{4} - \frac{26}{4} = \frac{-3}{4} \checkmark$$

- 13) Use an inequality symbol to write the statement $3x$ is between -3 and 6 including -3 and excluding 6 .

- Solution -

$$-3 \leq 3x < 6$$

- 14) Simplify by using the distributive property.

$$-7(2u - 4v + 3w)$$

- Solution -

$$-7(2u - 4v + 3w)$$

Distribute by multiplying -7 by each term inside the parenthesis.

$$= -14u + 28v - 21w \checkmark$$

15. In August, Alison Romike began with a checking account of \$ 871.47. Her checks and deposits for August are:

Checks	Deposits
\$35.68	\$83.55
\$23.26	\$120.95
\$5.91	

-Solution-

Add all the deposits to the checking account and Subtract the checks from the deposits:

$$871.47 + 83.55 + 120.95 - 35.68 \\ - 23.26 - 5.91 = \$1011.12$$

16. Use the indicated property to write a new expression that is equal to the given expression

$$(w+8) + (-4), \text{ Associative.}$$

- Solution -

Associative property for addition keeps the same terms in order and the only thing that changes is the parenthesis.

$$\text{Answer is: } w + [8 + (-4)]$$

To Simplify the expression, just combine like terms.

$$w + 4 \checkmark$$

17, The table shows the change in Consumer price indexes.

<u>Commodity</u>	<u>Change from 95 to 96</u>	<u>change from 96 to 97</u>
Apparel	- 0.9	0.2
Audio / Video Equipment	- 2.5	- 2.2

which one has a greater absolute value.
the change in apparel from 95 to 96
or 96 to 97 ?

- Solution -

$$\text{Change in apparel from 95 to 96} = |-0.9| = 0.9.$$

$$\text{Change in apparel from 96 to 97} = |0.2| = 0.2$$

0.9 is greater than 0.2.

Therefore, the change in apparel from 95 to 96 is greater. ✓

18. Simplify the expression:

$$-5 - (2 - 3p)$$

- Solution -

$$-5 - (2 - 3p)$$

Change all the signs that follow
the subtraction sign:

$$-5 - 2 + 3p$$

Combine like
terms.

$$-7 + 3p \checkmark$$

19. Average hourly earnings of production
workers at Nassbaum steel from 1996
to 2002 are approximated by:

$$y = 0.499x - 974.9$$

Where x represents the year and y represents
the hourly earnings.

Approximate the average hourly earnings
in 2000?

- Solution -

$$y = 0.499x - 974.9$$

Replace x with 2000.

$$y = 0.499(2000) - 974.9$$

$$= \$ 23.10 \checkmark$$

20, What is the matching equation for:
Sixteen minus eleven-fourth of a
number is 5.

- Solution -

$$16 - \frac{11}{4}x = 5$$

21, Simplify the expression:

$$4y^2 - 2y^3 - 7y^2 + 4y^3$$

- Solution -

Combine like terms:

$$\begin{aligned} 4y^2 - 7y^2 - 2y^3 + 4y^3 \\ = -3y^2 + 2y^3 \end{aligned}$$

22, Write in algebraic terms (in inequalities).

a, S is between 141 and 155

inclusive

- Solution -

$$141 \leq S \leq 155 \checkmark$$

b, X is between 14 and 19

inclusive

- Solution -

$$14 \leq X \leq 19 \checkmark$$

c, X is over 19

- Solution -

$$X > 19 \checkmark$$

~ Whole Numbers ~

- 1) State the digit for the given place in the number.

8,839,306,215

Ten thousands Thousands Hundreds Tens ones

and so on.

2,

option	Cost
7-passenger Seating	\$ 417
Anti lock brakes	\$ 589
Rear Window defroster	\$ 175

A car dealer offers an option Value package of 7-passenger seating, anti-lock brakes, and a rear-window defroster for \$1061. How much would a customer save by buying the Value package?

Solution -

Round: 417 to the nearest 100's = \$ 400
589 : 1 = 1 = 1 = \$ 600
175 : 1 = 1 = 1 = \$ 200

$$\text{Total} = 400 + 600 + 200 = 1200.$$

\$1061 is rounded to 1100

$$\text{Estimated Savings} = 1200 - 1100 = 100$$

Exact Saving.

$$417 + 589 + 175 = 1181$$

$$1181 - 1061 = \$120 \checkmark$$

3, Estimate by front end rounding.

$$4050 + 75 + 801 + 3878$$

- Solution -

4050 becomes 4000

75 becomes 100

801 becomes 800

3878 becomes 4000

Add the numbers $4000 + 100 + 800 + 4000$
 $= 8900 \checkmark$

4, Round 8,882,100,395 to the nearest billion.

- Solution -

882,100,395 will be rounded up

Answer is 9,000,000,000 \checkmark

5. Use the order of operation to simplify:

$$2 + 13 - 2\sqrt{9} + 4\sqrt{25} = 6 \cdot 2$$

- Solution -

$$\sqrt{9} = 3 \quad ; \quad \sqrt{25} = 5$$

Replace the answers in the given problem:

$$2 + 13 - 2 \times 3 + 4 \times 5 = 6 \cdot 2$$

Do multiplication first:

$$2 + 13 - 6 + 20 = 12$$

$$= 15 - 6 + 20 = 12$$

$$= 9 + 20 - 12 = 17 \checkmark$$

6.

Simplify: $6 \cdot 2^2 + \frac{0}{2}$

- Solution -

$$\frac{0}{2} = 0$$

$$6 \cdot 2^2 = 6 \cdot 4 = 24 \checkmark$$

7,

Find the $\sqrt{841}$

- Solution -

Use a calculator. Answer is 29 ✓

8,

Find the GCF of

60, 14, 35

Solution -

Write each number as a prime factor
and look for common numbers, then
multiply them.

$$60 = 2 \times 3 \times 2 \times 5$$

$$14 = 2 \times 7$$

$$35 = 5 \times 7$$

Since there is not a single number
in common, the answer is 1.

9,

a, Round 19,537 to the nearest
ten

- Solution -

37 becomes 40 \rightarrow 19540

b, Nearest 100: 537 becomes 500

Answer is 19500

c, Nearest 1000: 19537 becomes 20,000

10. The standard form of a number is 8.
a, Factors of repeated multiplication is?
-Solution-
 $8 = 2 \times 2 \times 2$

- b, Write 8 in exponential form:
 $8 = 2^3$

11. Identify the base & the exponent of

11^3
-Solution-
Base is 11.
Exponent is 3.
The expression simplified is:
 $11 \times 11 \times 11 = 1331$

12. Last month a nurse worked
thirteen 10 hour shifts and three
12-hour shifts. At \$23 per hour,
what was the nurse's total hourly
income before deductions.

Solution-

$$\text{Total hours worked} = 13 \times 10 + 3 \times 12 = 166 \text{ hours}$$

$$\text{Total income} = 166 \times 23 = \$3818$$

13) Find the Quotient Using short division Identify the dividend, the divisor and the Quotient. 5190

Solution-

$$\begin{array}{r} 38 \\ \hline 5 \overline{) 190 } \end{array}$$

Dividend is 190 ✓

Divisor is 5 ✓

Quotient is 38 ✓

14, find the Prime factorization of 2200 using exponents when repeated factors appear.

- Solution -

$$\begin{aligned} 2200 &= 22 \times 100 \\ &= 2 \times 11 \times 10 \times 10 \\ &= 2 \times 11 \times 2 \times 5 \times 2 \times 5 \\ &= 2^3 \times 5^2 \times 11 \end{aligned}$$

15,

Identify the number as prime, Composite or neither. If the number is composite, write it as the product of prime factors.

1155

- Solution -

Since 1155 ends with 5, you can divide it by 5.

$$= 5 \times 231$$

Now 231 is divisible by 3.

$$5 \times 3 \times 77$$

Now 77 is divisible by 11.

$$5 \times 3 \times 11 \times 7 \quad \checkmark$$

16,

A surgical technologist made \$40128 last year. He is paid twice a month. What is the gross total amount of each of his pay checks?

- Solution -

twice a month \rightarrow it means he gets paid $2 \times 12 = 24$ times a year.

$$24 \overline{)40128} \quad - \$1672 \quad \checkmark$$

17) Find the GCF of 24 and 56.

- Solution -

Write 24 as prime factors: $2 \times 2 \times 2 \times 3$

Write 56 as prime factors: $2 \times 2 \times 2 \times 7$

There are three 2's that are common.

Multiply them: $2 \times 2 \times 2 = 8$ ✓

18) Estimate $320,657$ to the nearest 100,000 and 6527 to the nearest 1000 and then multiply.

- Solution -

$320,657$ is rounded to $300,000$.

6527 is rounded to 7000

$$300,000 \times 7000 = 21,000,000,000$$

19,

Divide by using long division

$$28 \overline{)67344}$$

Find the Quotient and the remainder.

Solution -

by dividing 67344 by 28 by using the calculator, you get 2405.142

Keep 2405 and discard .142

$$\begin{array}{r} 2405 \\ 28 \overline{)67344} \end{array}$$

Multiply 2405 by
28

you get 67340

$$\begin{array}{r} 2405 \\ 28 \overline{)67344} \\ -67340 \\ \hline 4 \end{array}$$

Quotient = 2405 ✓

Remainder = 4 ✓

20)

A masonry contractor is preparing an estimate for building a stone wall and a gate. He estimates that the job will take a 40-hour work week.

He plans to have 3 laborers at \$15 per hour and 6 masons at \$24 per hour.

He'll need \$3243 worth of materials and wishes to make a profit of \$600. Write a

mathematical statement that will give the estimated cost of the job.

Then calculate this total.

-Solution -

$$3 \times 15 \times 40 + 6 \times 24 \times 40 + 3243 + 600$$

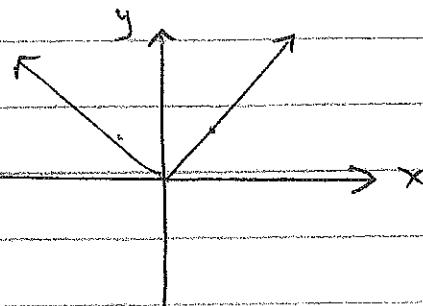
$$= 11403 \checkmark$$

Introduction To Functions

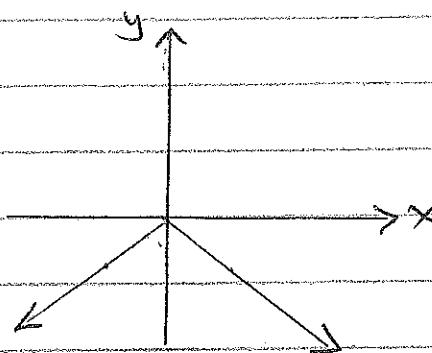
y Graph: $f(x) = -|x+3| - 6$

-Solution-

The graph of $f(x) = |x|$ looks like

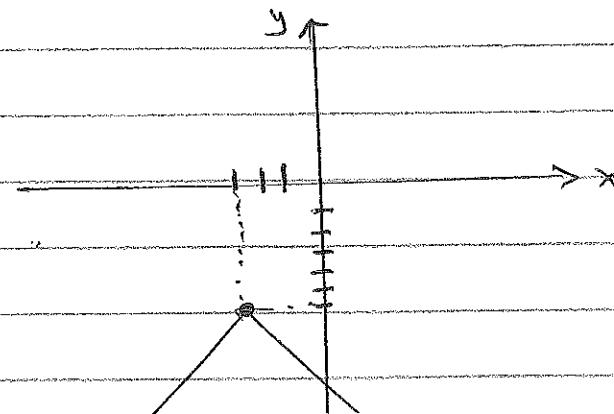


at the graph of $f(x) = -|x|$ looks like



To graph $f(x) = -|x+3| - 6$,

Move the vertex 3 units to the left
and 6 units down.



2. Inequalities with \geq or \leq

Use a solid line.

And inequalities $<$ or $>$ symbols
use a dashed line.

3. $f(x) = |x + 6|$

Find: a, $f(6)$

- Solution -

Replace x with 6 in $f(x)$

$$= |6 + 6| = |12| = 12 \checkmark$$

b, $f(-10) = ?$

- Solution -

Replace x with -10 in $f(x)$

$$= |-10 + 6| = |-4| = 4 \checkmark$$

c, $f(0) = ?$

- Solution -

Replace x with 0 in $f(x)$

$$= |0 + 6| = |6| = 6 \checkmark$$

4,

x	y
-1	-3
0	-2
1	-1
2	0

a, what is the slope?

Solution -

Pick any 2 points from the given table and label them as follows:

$$(-1, -3) \quad x_1, y_1$$

$$(0, -2) \quad x_2, y_2$$

$$m = \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - (-3)}{0 - (-1)}$$

$$= \frac{-2 + 3}{0 + 1} = \frac{1}{1} = 1 \checkmark$$

b, What is the equation of the line written in the form $y = mx + b$

Solution -

We already found m to be 1.

The value of b is the value of y when $x=0$ which is -2.

$$y = x - 2 \quad \checkmark$$

5) x is called the independent
 y is called the dependent.

6) Find the domain: $f(x) = \frac{8}{16x-7}$

- Solution -

The domain is all values of x
except the ones that make the
denominator $= 0$.

Set $16x-7 = 0$ and solve for x .

$$\begin{array}{rcl} 16x-7 & = & 0 \\ +7 & & +7 \end{array}$$

$$16x = 7 \Rightarrow x = 7/16.$$

Answer is: $(-\infty, 7/16) \cup (7/16, \infty)$ ✓

Fixed Cost	Variable Cost	Price of the item
\$1944	\$395	\$314

1

a) find the cost function.

Solution -

$$C_{Cx} = 1944 + 395 x$$

b) find the Revenue function:

- Solution -

$$R(x) = 314 \times$$

c Find the Profit function.

Solution

$$P(x) = R(x) = C(x)$$

$$= 314x - 1944 - 395x = -81x - 1944 \checkmark$$

d, find the break even point.

Solution -

Break-even \Rightarrow profit = 0. or $P(x) = 0$

$$-81x - 1944 = 0$$

+1944 +1944

Wavy

$$-81x = 1944 \implies$$

e. The restriction on Sale is 29 units

Make the right decision

Should the company produce?

- Solution -

Since the # of units is negative, the answer is "No"

8, $f(x) = |x - 8| + 7$

Find the intercepts.

- Solution -

X-intercepts: Replace y or $f(x)$ with 0 and solve for x .

$$0 = |x - 8| + 7$$

Subtract 7.

$-7 = |x - 8|$, since absolute value of a number can not be $(-)$, there is no x -intercept.

Y-intercept: Replace x with 0 in $f(x)$.

$$f(x) = y = |0 - 8| + 7$$

$$= |-8| + 7 = 8 + 7 = 15$$

$$(0, 15) \checkmark$$

a) Determine whether the following relation represents a function.

$$\{(7, 5), (-2, 6), (1, 1), (6, 6)\}$$

Solution -

Since the x value is not repeated,
it is a function ✓

b) What is the domain?

Solution -

Domain is the values of x .

$$\{7, -2, 1, 6\}$$

c) What is the range?

Solution -

Range is the values of y

(Do not duplicate the answers)

$$\{5, 6, 13\}$$

10, A company that manufactures bicycles has a fixed cost of \$2000. It costs \$200 to produce each bicycle. The total cost for the company is the sum of its fixed cost and variable costs.

a, find the cost function.

- Solution -

$$C(x) = 2000 + 200x$$

b, find $C(30)$.

- Solution -

Replace x with 30 in $C(x)$

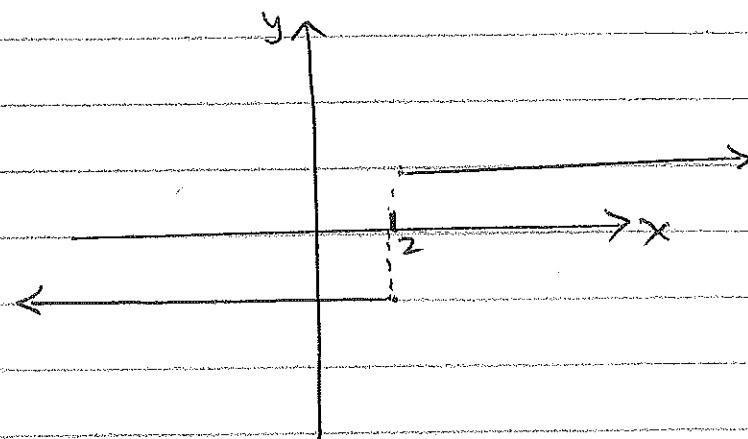
$$= 2000 + 200(30)$$

$$= 2000 + 6000 = \$8000$$

11) Sketch the graph.

a) $f(x) = \frac{|x-2|}{x-2}$

- Solution -



b) What is the domain?

- Solution -

$$(-\infty, 2) \cup (2, \infty)$$

c) What is the range?

- Solution -

$$\{-1, 1\}$$

d) Which statement best describes the function?

- Solution -

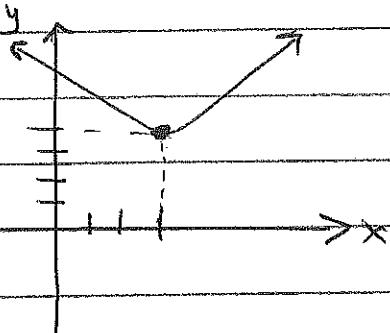
It is constant over the intervals.

$$(-\infty, 2) \text{ and } (2, \infty)$$

12, a, Graph: $f(x) = |x - 3| + 4$.

- Solution -

The vertex coordinates is $(3, 4)$.



b, Find the domain.

- Solution -

$$(-\infty, \infty)$$

c, Find the range.

- Solution -

$$[4, \infty)$$

13, Which graph illustrates a one-to-one function.

- Solution -

One-to-one function \rightarrow you can not have the same range for 2 different domains. straight lines are examples of one-to-one functions.

14,

Evaluate the function $f(x) = 5x + 5$
 at the given values of the independent
 variable & simplify.

a, $f(-6) = ?$

- Solution -
 Replace x with -6 in $f(x)$

$$= 5(-6) + 5 = -30 + 5 = -25 \checkmark$$

b, find $f(x+2)$

- Solution -
 Replace x with $x+2$ in $f(x)$.

$$\begin{aligned} &= 5(x+2) + 5 = 5x + 10 + 5 \\ &= 5x + 15 \checkmark \end{aligned}$$

c, find $f(-x)$.

- Solution -
 Replace x with $-x$ in $f(x)$

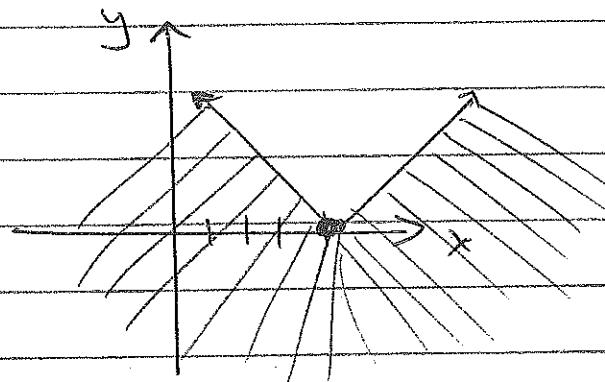
$$\begin{aligned} &= 5(-x) + 5 \\ &= -5x + 5 \checkmark \end{aligned}$$

15) Graph: $y \leq |x - 4|$

- Solution -

First graph $y = |x - 4|$
Vertex is $(4, 0)$.

Then \leq \rightarrow solid lines and
the shades have to be under the
graph.

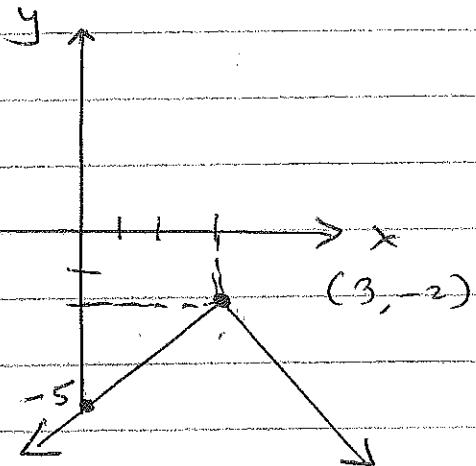


16) Which of the following is the
correct way to write a
linear function?

- Solution -

$$f(x) = mx + b \checkmark$$

17)



a, What is the function's domain?

- Solution -

$$(-\infty, \infty)$$

b, What is the function's range?

- Solution -

$$(-\infty, -2]$$

c, Find the x-intercepts.

- Solution -

Since the graph does not cross the x-axis,
there is no x-intercept.

d, Find the y-intercept.

- Solution -

It crosses the y-axis at $(0, -5)$

e, find $f(0)$

- Solution -

When $x=0$, $y = -5$. (read from graph)

18,

In 1997, 69% of students at a local college regularly used the library for their internet services. This percentage has decreased at an average rate of approximately 1.1 each year since then.

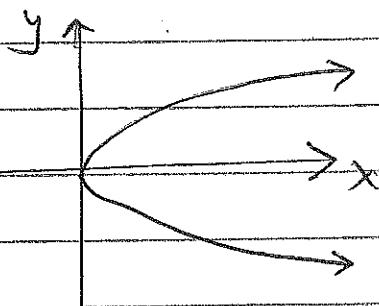
Find a linear function in slope-intercept form that models the percentage of students, $P(x)$, who regularly used the library for internet service x years after 1997.

Solution -

$$P(x) = 69 - 1.1x$$

19,

Determine whether the graph is that of a function.



Solution -

If you draw a vertical line on the graph, it will cross the graph in 2 points and not one. Therefore, it is NOT a function.

20,

Determine whether the following equation defines y as a function of x .

$$x + y = 18$$

Solution -

Since it is a linear function, the answer is "yes" "y" is a function of x .

21,

Sketch the graph of the linear function, then give its domain and range.

$$f(x) = \frac{1}{4}x + 3$$

- Solution -

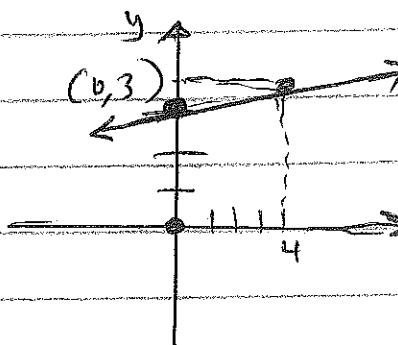
y -intercept is $(0, 3)$.

$$\text{slope} = \frac{1}{4} \leftarrow \begin{matrix} \text{Rise} \\ \text{Run} \end{matrix}$$

Start by plotting $(0, 3)$ and from that point go up 1 and move horizontally 4 units to the right.

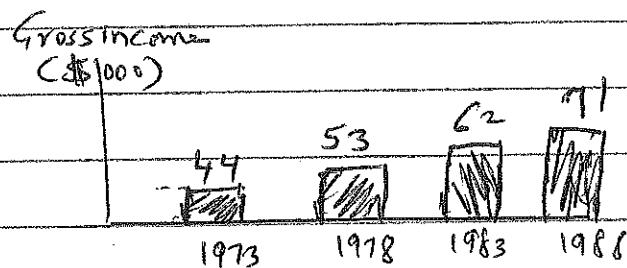
$$\text{Domain: } (-\infty, \infty)$$

$$\text{Range: } (-\infty, \infty)$$



23

Write the information on the chart as a set of ordered pairs and determine if the set defines a function.



- Solution -

Order pairs are:

$$\{(1973, 44), (1978, 53), (1983, 62), (1988, 71)\}$$

It does represent a function because the x values are not duplicated.

Linear Equations +
~ Inequalities ~

1) Solve the absolute value inequality & graph the solution set:

$$|-4x + 12| \leq 3$$

- Solution -

$$\begin{array}{c} -3 \leq -4x + 12 \leq 3 \\ \quad \quad \quad -12 \quad \quad \quad -12 \\ \hline -15 \leq -4x \leq -9 \end{array}$$

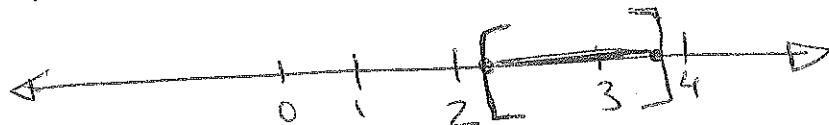
Divide each term by -4.

$$\frac{-15}{-4} \leq \frac{-4x}{-4} \leq \frac{-9}{-4}$$

$$\frac{15}{4} \leq x \leq \frac{9}{4} \quad \text{Swap the numbers.}$$

$$\frac{9}{4} \leq x \leq \frac{15}{4} \checkmark$$

Graph. $\frac{9}{4}$ is 2.25 \rightarrow $15/4$ is 3.75.



2.

Solve : $\left| \frac{2x-6}{4} \right| > 8$

- Solution -

Rewrite: $\frac{2x-6}{4} > \frac{8}{1}$ cross multiply

$$\begin{aligned} 2x-6 &> 32 \\ \underbrace{+6 \quad +6}_{2x > 38} &\implies x > 19 \checkmark \end{aligned}$$

Now set. $\frac{2x-6}{4} < -\frac{8}{1}$ cross multiply

$$2x-6 < -32 \quad \text{Add 6 to both sides.}$$

$$2x < -26$$

$$x < -13 \checkmark$$

Writing both answers in interval form

$$(-\infty, -13) \cup (19, \infty) \checkmark$$

3. Solve the inequality.

$$7(5g - 8) - 10g < 25g + 7$$

- Solution -

Distribute first:

$$35g - 56 - 10g < 25g + 7$$

Combine like terms:

$$25g - 56 < 25g + 7$$

Subtract $25g$ from both sides.

$$-56 < 7 \quad (\text{True})$$

Answer is all real numbers: $(-\infty, \infty)$.

←→
The graph is shaded all the way.

4. Solve: $11(t - 3) + 4t = 5(3t + 3) - 9$

- Solution -

Distribute first:

$$11t - 33 + 4t = 15t + 15 - 9$$

Combine like terms.

$$15t - 33 = 15t + 6$$

Subtract $15t$ from both sides.

$$-33 = 6$$

(Not true)
No solution.

5. Solve: $2x - 2 > -14$

or

$$3x + 1 \leq 19$$

- Solution -

$$2x - 2 > -14 \quad \text{Add } 2 \text{ to both sides}$$

$$2x > -12 \quad \Rightarrow \quad x > -6 \checkmark$$

Solve for x in:

$$\begin{array}{r} 3x + 1 \leq 19 \\ -1 \\ \hline 3x \leq 18 \end{array} \quad \Rightarrow \quad x \leq 6 \checkmark$$

Answer is all real numbers

$(-\infty, \infty)$



shaded all the way

6. Solve: $6x - 8 < -20$ and $-3x + 1 > -8$

- Solution -

$$6x - 8 < -20 \quad \text{Add } 8 \text{ to both sides}$$

$$6x < -12 \quad \Rightarrow \quad x < -2 \checkmark$$

$-3x + 1 > -8$ Subtract 1 from both sides

$$-3x > -9 \quad \text{Divide by } -3$$

$$x < 3 \checkmark$$

Answers: $x < -2$ and $x < 3$

Take the common
 $x < -2$



solution which is

7, Translate the words in to an equation:

Twice a number is equal to 8 more than 5 times the number.

- Solution -

$$2x = 8 + 5x \checkmark$$

8, A race car driver won a 500 mile race with a speed of 169.2 miles per hour. Find the driver's time & round to the nearest thousandth.

- Solution -

$$t = \frac{d}{s} = \frac{500}{169.2} = 2.955 \checkmark$$

9, Solve: $0.4(x+3) - 0.7(x+3) = -0.3x - 0.9$

- Solution -

Distribute first:

$$0.4x + 1.2 - 0.7x - 2.1 = -0.3x - 0.9$$

Combine like terms:

$$-0.3x - 0.9 = -0.3x - 0.9$$

Since the left side of the equation is the same as the right side, then the solution is "All real numbers"

10. Solve for y :

$$x = \frac{3y - x}{y}$$

- Solution -

Cross multiply:

$$xy = 3y - x \quad \text{Move } 3y \text{ to the left side of the equation.}$$

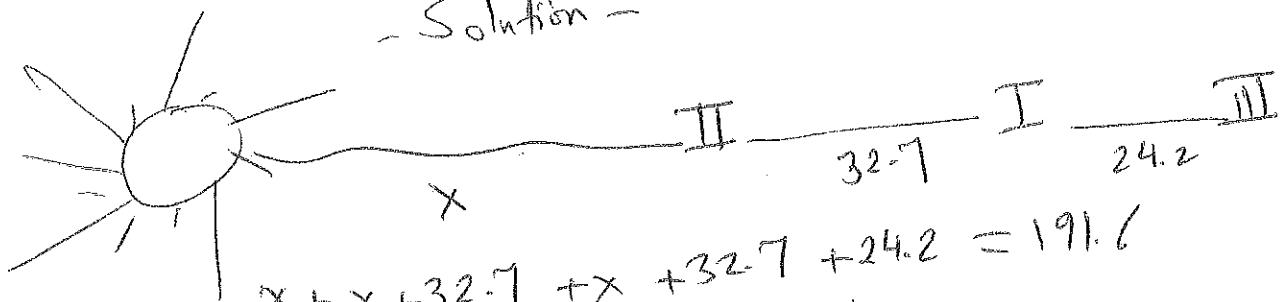
$$xy - 3y = -x \quad \text{Take "y" as a common factor.}$$

$$y(x - 3) = -x$$

$$\text{Therefore } y = \frac{-x}{x - 3} \checkmark$$

11. If planet I is 32.7 miles (million) farther from the sun than planet II, then planet III is 24.2 miles (million) farther from the sun than planet I. When the total distances for these 3 planets from the Sun is 191.6 million miles. How far away from the Sun is planet III?

- Solution -



$$x + x + 32.7 + x + 32.7 + 24.2 = 191.6$$

$$3x + 89.6 = 191.6$$

$$3x = 102 \quad \Rightarrow x = 34 \checkmark$$

12. Solve: $\frac{3x}{2} - x = \frac{x}{6} - \frac{2}{3}$

Solution -

Multiply each term by 6.

$$6 \cdot \frac{3x}{2} - 6 \cdot x = 6 \cdot \frac{x}{6} - 6 \cdot \frac{2}{3}$$

$$9x - 6x = x - 4$$

$$3x = x - 4$$

$$2x = -4 \implies x = -2 \checkmark$$

13. The body mass index I is used to determine an individual risk for heart disease. An index less than 25 indicates a low risk. The body mass index is given by the formula on model.

$$I = \frac{700W}{H^2}$$

Where W = weight in pounds, and H = height in inches. Jerome is 76 inches tall. What weights will keep his body mass index between 25 and 33.

Solution -

$$\text{for } I = 25 \quad \text{and } H = 76$$

$$25 = \frac{700W}{76^2}$$

$$\frac{25}{1} = \frac{700W}{5776}$$

Cross multiply:

$$700W = 144400 \Rightarrow W = 206$$

for $I = 33$ & $H = 76$.

$$\cancel{33} \quad \frac{33}{?} = \frac{700W}{76} = \frac{700W}{5776}$$

Cross multiply: $700W = 190608$

$$W = 272$$

$$206 < W < 272 \checkmark$$

14. Solve + graph: $12x + 2 > 2(4x+1) - x + 9$
- Solution -

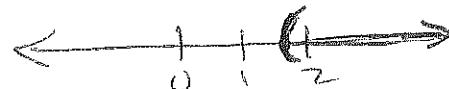
Distribute first:

$$12x + 2 > 8x + 2 - x + 9. \quad \text{Combine like terms:}$$

$$12x + 2 > 7x + 11. \quad \text{Subtract } 7x \text{ from both sides}$$

$$\cancel{12x} \quad 5x + 2 > 11 \Rightarrow 5x > 9$$

$$x > \frac{9}{5}$$



15, The recommended daily intake (RD) of a nutritional supplement for a certain age group is 500 mg / day. Actually, supplement needs vary from person to person. Write an absolute value inequality to express the RD plus or minus 90 mg and solve it.

- Solution -

$$|x - 500| \leq 90$$

$$-90 \leq x - 500 \leq 90$$

$$+500 \quad +500 \quad +500$$

$$\underbrace{\hspace{10em}}_{410 \leq x \leq 590}$$

16, Solve : $|7n + 3| + 11 = 4$

- Solution -

$$|7n + 3| + 11 = 4$$

$$\underbrace{\hspace{10em}}_{|7n + 3| = -7}$$

since absolute value of a number can not be negative. \Rightarrow Solution is empty set of "No solution"

17. Chris can be paid in 1 of 2 ways.

Plan A is a salary of \$460 /month plus a commission of 7% of sales. Plan B is a salary of \$682 per month plus a commission of 4% of sales. For what amounts of sales is Chris better off selecting plan A?

- Solution -

$$460 + 0.07x > 682 + 0.04x$$

Subtract 0.04x from both sides.

$$460 + 0.03x > 682$$

Subtract 460 from both sides.

$$0.03x > 222$$

$$x > 7400 \checkmark$$

18. The value of an uncirculated "Mint-State - 65"

1950 Jefferson Nickel minted in Denver is $\frac{7}{4}$ the value of a 1945 nickel minted in Philadelphia in similar condition. Together the

\$77.

Total value of the 2 coins is

- Solution -

Let 1945 nickel price = x
 $\therefore 1950 \text{ nickel price} = \frac{1}{4}x$

$$x + \frac{1}{4}x = 77 \quad \text{Multiply each term by 4.}$$

$$4x + 4 \cdot \frac{1}{4}x = 4(77)$$

$$4x + 7x = 308 \Rightarrow 11x = 308$$

$$x = 28 \checkmark$$

$$1950 \text{ nickel} = \frac{1}{4}x = \frac{1}{4} \times 28 = \$49 \checkmark$$

19. Solve. $-3(7p + 6) - 4 + 40p = 27 + 18p$
- Solution -

Distribute first:

$$-21p - 18 - 4 + 40p = 27 + 18p$$

Combine like terms:

$$19p - 22 = 27 + 18p$$

Subtract $18p$ from
both sides

$$p - 22 = 27 \Rightarrow p = 49 \checkmark$$

20, Simplify the expression:
 $-6.7(3S + 6) - 1.8(2S - 5)$
- Solution -

Distribute first:

$$-20.1S - 40.2 - 3.6S + 9$$

Combine like terms:

$$-23.7S - 31.2 \checkmark$$

21, The product of 4 and 7 more than a number. "Write an algebraic expression".
- Solution -

$$4(x + 7) \checkmark$$

22, Solve. $|2y - 3| = |27 - 3y|$
- Solution -

Drop the absolute value bars:

$$2y - 3 = 27 - 3y \quad \text{Add } 3y \text{ to both sides}$$

$$5y - 3 = 27$$

$$\Rightarrow 5y = 30 \quad \therefore y = 6 \checkmark$$

Drop the bars now and change the signs of
 $27 - 3y$

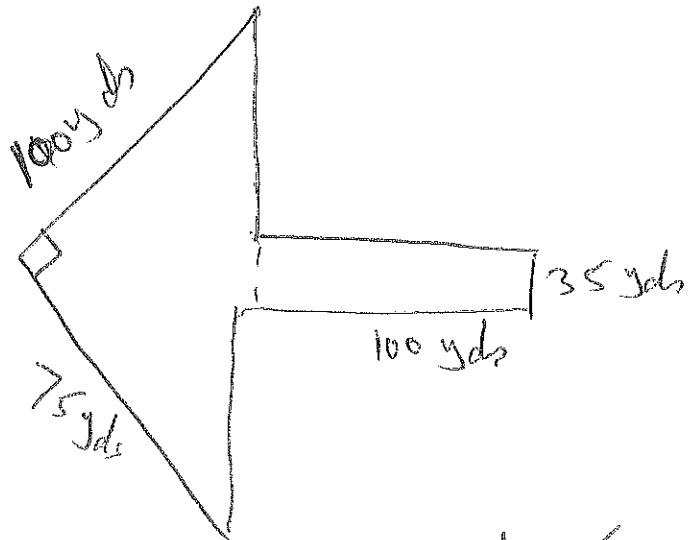
$$2y - 3 = -27 + 3y$$

Subtract $3y$ from both sides

$$-y - 3 = -27 \Rightarrow -y = 24 \Rightarrow y = -24$$

Answer is 6, -24

23.



Distance around
the streets

$$= 100 \text{ yd} + 75 \text{ yd} = 175 \text{ yds} \checkmark$$

Area of the lot = Area of the
triangle + Area of the rectangle

$$\text{Area of the triangle} = \frac{100 \times 75}{2} = 3750 \text{ yd}^2$$

$$\text{Area of the rectangle} = 100 \times 35 = 3500 \text{ yd}^2$$

$$\begin{aligned}\text{Total area} &= 3750 + 3500 \\ &= 7250 \text{ yd}^2 \checkmark\end{aligned}$$

Exponents and Polynomials

- b) The total imports of Petroleum (in millions of barrels per day) in a given year is approximated by:

$$38x^3 - 523x^2 + 2087x + 6058,$$

where x represents the number of years since 1975. The corresponding total of exports of Petroleum is:

$$-x^3 + 10x^2 + 27x + 18$$

Write a polynomial that represents how many more barrels per day were imported than exported in a given year.

Solution -

$$\text{Imported} - \text{Exported} =$$
$$(38x^3 - 523x^2 + 2087x + 6058) -$$

$$(-x^3 + 10x^2 + 27x + 18)$$

change all the signs of the terms that follow the subtraction sign:

$$38x^3 - 523x^2 + 2087x + 6058 + x^3 - 10x^2 - 27x - 18.$$

Now combine like terms:

$$39x^3 - 533x^2 + 2060x + 5873$$

3. Use the exponent rule to simplify the expression:

$$\frac{(m^8 n)^{-4}}{m^{-17} n^5}$$

Rule is: $(a^b)^c = a^{b \times c}$

$$(m^8 n)^{-4} = m^{8 \times -4} n^{-4} = m^{-32} n^{-4}$$

Replace the answer in the given problem:

$$\frac{m^{-32} n^{-4}}{m^{-17} n^5}$$

Rule: $\frac{x^a}{x^b} = x^{a-b}$

$$\frac{m^{-32} n^{-4-5}}{m^{-17}} = m^{-32+17} n^{-9}$$
$$= m^{-15} n^{-9}$$

Rule: $x^{-a} = \frac{1}{x^a}$

Therefore, the answer is

$$\frac{1}{m^{15} n^9} \checkmark$$

3) Convert the number 1525000000 to scientific notation.

- Solution -

A number is in scientific notation if it is between 1 and 10; could equal 1, but not 10.

1.525×10^{10} since we have to move the decimal from right to left 10 places.

4) Find the product:

$$x(3x - 5)(x + 5)$$

- Solution -

Multiply $(3x - 5)(x + 5)$ first:

$$= 3x^2 + 15x - 5x - 25$$

$$= 3x^2 + 10x - 25 \checkmark$$

Now, multiply the result by x :

$$x(3x^2 + 10x - 25)$$

$$= 3x^3 + 10x^2 - 25x \checkmark$$

5, find the product:

$$6x^3(3x^2 - 6x + 3)$$

- Solution -

Rule: $x^a \cdot x^b = x^{a+b}$.

Distribute:

$$6x^3(3x^2 - 6x + 3)$$

$$= 18x^5 - 36x^4 + 18x^3 \checkmark$$

6, Divide: $\frac{21x^9 - 21x^7 + 35x^2}{-7x^2}$

- Solution -

Rule: $\frac{x^a}{x^b} = x^{a-b}$.

Divide each numerator by $-7x^2$.

$$\frac{21x^9}{-7x^2} - \frac{21x^7}{-7x^2} + \frac{35x^2}{-7x^2}$$

$$= -3x^7 + 3x^5 - 5 \checkmark$$

7, a, classify the polynomial.

$$x^2 - 20x + 100$$

- Solution -

Since it has 3 terms, it is a trinomial ✓

b, What is the degree?

- Solution -

Since the largest exponent is 2 (x^2), the degree is 2 ✓

8, Evaluate the polynomial for $x = -1$.

$$-3x^3 + 7x^2 - 4x - 2$$

- Solution -

Replace x with -1 .

$$-3(-1)^3 + 7(-1)^2 - 4(-1) - 2$$

$$= -3(-1)(-1)(-1) + 7(1) + 4 - 2$$

$$= 3 + 7 + 4 - 2 = 12 \checkmark$$

9. Use the distributive property to find the product.

$$(y-7)(-4y^2 + 6y + 3)$$

- Solution -

$$(y-7)(-4y^2 + 6y + 3)$$

Multiply y by $(-4y^2 + 6y + 3)$ first:

$$= -4y^3 + 6y^2 + 3y \checkmark$$

Now multiply -7 by $(-4y^2 + 6y + 3)$

$$= 28y^2 - 42y - 21 \checkmark$$

Combine both results:

$$-4y^3 + 6y^2 + 3y + 28y^2 - 42y - 21$$

Combine like terms:

$$-4y^3 + 34y^2 - 39y - 21 \checkmark$$

10) Divide:

$$\frac{4x^4 - 22x^3 + 11x^2 - 7x + 10}{x - 5}$$

Solution —

$$\begin{array}{r} 4x^3 - 2x^2 + x - 2 \\ \hline x - 5 \left| \begin{array}{r} 4x^4 - 22x^3 + 11x^2 - 7x + 10 \\ -4x^4 + 20x^3 \\ \hline -2x^3 + 11x^2 \\ + 2x^3 - 10x^2 \\ \hline x^2 - 7x \\ - x^2 + 5x \\ \hline -2x + 10 \\ + 2x - 10 \\ \hline 0 \end{array} \right. \end{array}$$

← Remainder

Answer is

$$4x^3 - 2x^2 + x - 2 \checkmark$$

11) Simplify:

$$(5x^4y^2z)^3 \cdot (x^5y)^4$$

- Solution -

Rule: $(x^a)^b = x^{a+b}$

$$(5x^4y^2z)^3 = 5^3 x^{12} y^6 z^3.$$

$$(x^5y)^4 = x^{20} y^4.$$

Multiply both results:

$$5^3 x^{12} y^6 z^3 \cdot x^{20} y^4$$

Rule: $x^a \cdot x^b = x^{a+b}.$

$$= 125 x^{32} y^{10} z^3 \checkmark$$

12. Simplify: $\frac{(3u^2v^2)^{-4}(4u^{-2}v^3)^3}{(3u^2v^{-3})^{-2}}$

Rule: $(x^a)^b = x^{ab}$

multiply the exponents inside the parenthesis by the outside ones.

$$\frac{3^{-4} u^{-8} v^{-8} \cdot 4^3 u^{-6} v^9}{3^{-2} u^{-4} v^6}$$

Rule: $\frac{x^a}{x^b} = x^{a-b}$

$$\frac{3^{-4}}{3^{-2}} = 3^{-4-(-2)} = 3^{-2} \checkmark$$

$$\frac{u^{-8} \cdot u^{-6}}{u^{-4}} = \frac{u^{-14}}{u^{-4}} = u^{-14-(-4)} = u^{-10} \checkmark$$

$$\frac{v^{-8} \cdot v^9}{v^6} = \frac{v^{-1}}{v^6} = v^{-1-6} = v^{-5}$$

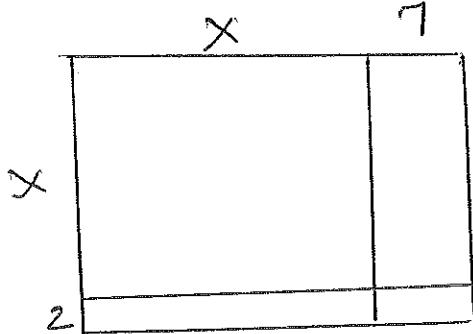
$$4^3 = 4 \times 4 \times 4 = 64.$$

$$3^{-2} \cdot u^{-10} \cdot v^{-5} \cdot 64$$

Rule: $x^{-a} = \frac{1}{x^a}$

$$\frac{1}{3^2 u^{10} v^5} = \frac{64}{9 u^{10} v^5} \checkmark$$

13 Find the area of the figure:



(length = $x + 7$)
width = $x + 2$

- Solution -

$$\begin{aligned} \text{Area} &= l \times w = (x + 7)(x + 2) \\ &= x^2 + 2x + 7x + 14 \\ &= x^2 + 9x + 14 \end{aligned}$$

14 Multiply: $(-3x^6)(-4x^3)$

- Solution -

Multiply the numbers first:

$$(-3)(-4) = 12$$

$$x^6 \cdot x^3 = x^9$$

Answer is $12x^9$

15, Find the product:

$$(4e + \frac{1}{5}f)^2$$

- Solution -

Rule is: $(a+b)^2 = a^2 + 2ab + b^2$.

$$(4e)^2 = 16e^2.$$

$$2(4e)(\frac{1}{5}f) = \frac{8}{5}f.$$

$$(\frac{1}{5}f)^2 = \frac{1}{25}f^2.$$

$$\text{Answer is: } 16e^2 + \frac{8}{5}f + \frac{1}{25}f^2$$

16, If $f(x) = (5x - 3)$ and $g(x) = (x - 5)$

Find $(fg)(x)$

- Solution -

Multiply both functions by each other.

$$(5x - 3)(x - 5)$$

$$= 5x^2 - 3x - 25x + 15$$

Combine like terms:

$$5x^2 - 28x + 15$$

17, Multiply:

$$(3t^2 - t)(3t^2 + 3t - 1)$$

- Solution -

First multiply $3t^2(3t^2 + 3t - 1)$

$$= 9t^4 + 9t^3 - 3t^2 \checkmark$$

Now multiply $-t(3t^2 + 3t - 1)$

$$= -3t^3 - 3t^2 + t \checkmark$$

Combine the results:

$$\begin{aligned} & 9t^4 + 9t^3 - 3t^2 - 3t^3 - 3t^2 + t \\ & = 9t^4 + 6t^3 - 6t^2 + t \checkmark \end{aligned}$$

18, $f(x) = 4x^2 + 7x - 5$

$$g(x) = -6x^2 + 3x - 20$$

a, find $(f+g)(x)$

- Solution -

Add both functions:

$$4x^2 + 7x - 5 - 6x^2 + 3x - 20$$

Combine like terms:

$$-2x^2 + 10x - 25 \checkmark$$

b, find $(f-g)(x)$

- Solution -

$$(4x^2 + 7x - 5) - (-6x^2 + 3x - 20)$$

Change all the signs that follow the subtraction.

$$\begin{aligned} & 4x^2 + 7x - 5 + 6x^2 - 3x + 20 \\ & = 10x^2 + 4x + 15 \checkmark \end{aligned}$$

19) Evaluate: $-3^0 + 3^0$

- Solution -

Rule: $x^0 = 1$.

any number raised to the power 0 = 1.

$$-3^0 + 3^0 = -1 + 1 = 0 \checkmark$$

20, Perform the indicated operations. Give the answer in scientific notation.

$$\frac{6.4 \times 10^{-4} \times 4.0 \times 10^{-3}}{2 \times 10^4 \times 3.2 \times 10^{-2}}$$

- Solution -

First work on the numerator:

$$6.4 \times 10^{-4} \times 4.0 \times 10^{-3} = 25.6 \times 10^{-7}$$

Now work on the denominator:

$$2 \times 10^4 \times 3.2 \times 10^{-2} = 6.4 \times 10^2$$

Now divide the numerator by the denominator:

$$\frac{25.6 \times 10^{-7}}{6.4 \times 10^2} = 4 \times 10^{-9} \quad (\text{remember}$$

the rule:

$$\frac{x^a}{x^b} = x^{a-b})$$

21) Combine like terms:

$$6 + 2a - (3a + 8) - (4a + 5)$$

- Solution -

Change the signs of all the terms that follow subtraction.

$$6 + 2a - 3a - 8 - 4a - 5.$$

Combine like terms:

$$= -7 - 5a \checkmark$$

22, Perform the operation:

$$(-2m^2 + 6n^2 - 10n) - [(6m^2 - 10m + 7) + (-6m^2 + 4n^2)]$$

- Solution -

Change all the signs of the terms that follow [

$$-2m^2 + 6n^2 - 10n - 6m^2 + 10m - 7 + 6m^2 - 4n^2.$$

Combine like terms:

$$= -2m^2 + 2n^2 - 10n + 10m - 7 \checkmark$$

23, Perform the indicated operations and give the answer in scientific notation.

$$\frac{6.4 \times 10^{-1} \times 4.5 \times 10^{-2}}{2 \times 10^2 \times 3.6 \times 10^{-4}}$$

- Solution -

Work on the numerator first:

$$6.4 \times 10^{-1} \times 4.5 \times 10^{-2} \quad \Rightarrow$$

multiply 6.4×4.5 , you get = 28.8

Multiply $10^{-1} \times 10^3$, you get 10^{-4}

Therefore the numerator result is:

$$28.8 \times 10^{-4}$$

Work on the denominator:

$$2 \times 10^2 \times 3.6 \times 10^{-4}$$

Multiply $2 \times 3.6 = 7.2$

and $10^2 \times 10^{-4} = 10^{-2}$,

therefore, the denominator result is:

$$7.2 \times 10^{-2}$$

Now divide the numerator result by the denominator result:

$$\frac{28.8 \times 10^{-4}}{7.2 \times 10^{-2}}$$

Divide 28.8 by 7.2, you get 4.

Divide 10^{-4} by 10^{-2} , you get 10^{-2}

Answer is 4×10^{-2} .

24, what polynomial, when divided by $8y^2$,
yields $3y^2 - 9y + 3$ as a Quotient?

- Solution -

Multiply $8y^2$ by $(3y^2 - 9y + 3)$.

$$8y^2(3y^2 - 9y + 3). \quad \text{Distribute.}$$

$$24y^4 - 72y^3 + 24y^2 \checkmark$$

25, Simplify: $(4x^3y^3z)^3 \cdot (x^3y)^5$.

- Solution -

Rule: $(x^a)^b = x^{a \cdot b}$

$$\text{So } (4x^3y^3z)^3 = 4^3 x^9 y^9 z^3 = (4x^9y^9z^3)$$

$$+ (x^3y)^5 = x^{15}y^5$$

Now multiply the terms by each other.

(Rule: $x^a \cdot x^b = x^{a+b}$)

$$64x^9y^9z^3 \cdot x^{15}y^5$$

$$= 64x^{24}y^{14}z^3 \checkmark$$

26,

$$(2g - h)^3 =$$

Solution -

Formula is $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$$(2g - h)^3 ; a = 2g \\ b = h$$

Replace a with "2g" and b with "h"

$$= (2g)^3 - 3(2g)^2(h) + 3(2g)(h)^2 - (h)^3 \\ = 8g^3 - 3(4g^2)h + 6gh^2 - h^3$$

$$= 8g^3 - 12g^2h + 6gh^2 - h^3 \checkmark$$

27, Multiply: $(2x+3)(2x-8)$

- Solution -

$$(2x+3)(2x-8) =$$

Multiply $2x$ by $(2x-8)$ first.

$$2x(2x-8) = \underline{4x^2-16x}$$

Now multiply

$$+3 \text{ by } (2x-8)$$

$$+3(2x-8) = +\underline{6x-24}$$

Combine both results:

$$4x^2-16x+6x-24$$

Combine like terms:

$$4x^2-10x-24$$

28, In 2005, the total waste generated in a certain country was 8.523×10^{11} pounds. Also in 2005, the country's population was 4.23×10^8 people. Determine the garbage per capita (per person) in that country in the year 2005.

- Solution -

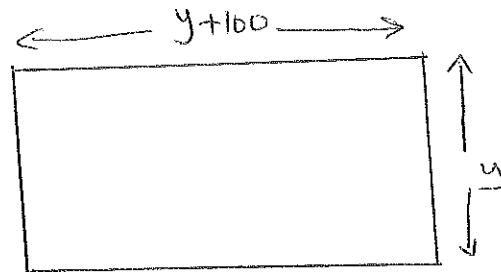
Divide 8.523×10^{11} by 4.23×10^8 .

Divide 8.523 by 4.23 first, you get 2.01

Now, Divide 10^{11} by 10^8 , you get 10^3 .

Answer is 2.01×10^3

29. The length and width of the rectangle in the figure differ by 100 mm. Find the area of the rectangle in terms of y without using Parenthesis.



Solution -

$$y(y + 100) \quad . \quad \text{Distribute}$$

$$y^2 + 100y \checkmark$$

~ Factoring Polynomials ~

1) Solve the equation.

$$y^3 - 13y^2 + 30y = 0$$

Solution -

$$y^3 - 13y^2 + 30y = 0$$

Take y as a common factor.

$$y(y^2 - 13y + 30) = 0$$

$$\text{Now factor } y^2 - 13y + 30 = (y-10)(y-3)$$

$$y(y-10)(y-3) = 0$$

Set each factor to 0 and solve for y .

$$y = 0 \checkmark$$

$$y-10=0 \Rightarrow y=10\checkmark$$

$$y-3=0 \Rightarrow y=3\checkmark$$

3 If a trinomial has a negative coefficient for the squared term, it's usually easier to factor by first factoring out the common factor -1. Use this method to factor.

$$-5a^2 - 26ab - 5b^2$$

$$= -1(5a^2 + 26ab + 5b^2)$$

$$= -1(5a+b)(a+5b)\checkmark$$

3, Factor the trinomial by grouping.

$$15m^2 - 7m - 4$$

- Solution -

Multiply 15 by -4, you get -60

Now think of 2 numbers, the product = -60,
and the sum = -7, the numbers are:

$$-12, +5$$

$$(15m - 12)(15m + 5)$$

Divide $15m - 12$ by 3

$$(5m - 4).$$

Divide $(15m + 5)$ by 5

$$(3m + 1).$$

Answer is $(5m - 4)(3m + 1)$

4, The dimensions of a rectangle are such that its length is 3 inches more than its width.

If the length were doubled and if the width were decreased by 1 in, the area would be increased by 50 in^2 . What are the length and width of the rectangle?

- Solution -

$$\text{Area} = (3+w)w - 3w + w^2 \quad w \quad | \quad w$$
$$2(3+w) \quad l = 3+w$$

$$\text{Area} = 2(3+w)(w+1) \quad | \quad w+1$$

$$A = 2(3+w)(w-1)$$

$$= 2(w^2 + 2w - 3)$$

$$= 2w^2 + 4w - 6 = 3w + w^2 + 50.$$

Make the equation = 0.

$$2w^2 + 4w - 6 - 3w - w^2 - 50 = 0.$$

Combine like terms.

$$w^2 + w - 56 = 0$$

Factor:

$$(w+8)(w-7) = 0$$

$$\Rightarrow w = -8 \times \quad w = 7 \checkmark$$

$$l = 3+w = 3+7 = 10\checkmark$$

5, Factor by grouping:

$$1 - f + fw - w$$

Solution -

$$\begin{aligned} & (1-f) + (fw-w) \\ & = (1-f) + w(f-1) \\ & = (f-1)(-1+w) \checkmark \end{aligned}$$

5, Factor: $125r^3 + 1$.

Solution -

$$\text{formula: } a^3 + b^3 = (a+b)(a^2 - ab + b^2).$$

$$125r^3 + 1 = (5r)^3 + (1)^3$$

$a = 5r$ and $b = 1$. Substitute.

$$(5r+1)(25r^2 - 5r + 1) \checkmark$$

7, Factor the trinomial:

$$c^2f^2 + 12cf + 27$$

Solution -

$$\text{Product} = 27$$

Sum = +12 \Rightarrow numbers are +9, +3.

$$(cf+9)(cf+3) \checkmark$$

8) Solve: $c^2 = -24 - 11c$

- Solution -

Make the equation = 0.

$$c^2 + 11c + 24 = 0 \quad \text{Factor it:}$$

$$(c+8)(c+3) = 0$$

Set each factor to 0 and solve for c.

$$c = -8, c = -3.$$

9) Factor the following trinomial:

$$t^2 + \frac{1}{4}t + \frac{1}{64}$$

- Solution -

$$\text{Product} = \frac{1}{64}, \quad \text{Sum} = \frac{1}{4}$$

numbers are $\frac{1}{8}, \frac{1}{8}$

$$(t + \frac{1}{8})^2$$

10) Solve: $3w^2 = 9w$

- Solution -

Make the equation = 0.

$$3w^2 - 9w = 0 \quad \text{Factor:}$$

$$3w(w-3) = 0 \Rightarrow w = 0 \checkmark$$

$$w = 3 \checkmark$$

- 11) Decide which is the correct factored form of the given polynomial.
- A, $(7x+2)(x-3)$
B, $(7x-2)(x+3)$
- $7x^2 - 19x - 6$

Solution -

Try choice "A"

$$(7x+2)(x-3)$$

$$= 7x^2 + 2x - 21x - 6$$

$$= 7x^2 - 19x - 6 \checkmark$$

12. Factor out the greatest common factor.

$$(8z-3)(z+7) \quad (8z-3)(z-6)$$

- Solution -

$$(8z-3)(z+7) - (8z-3)(z-6)$$

$$= (8z-3)(z+7 - (z-6))$$

$$= (8z-3)(z+7 - z + 6)$$

$$= (8z-3)(13) \checkmark$$

13, The sum of the squares of 2 consecutive odd positive integers is 130. Find the integers.

- Solution -

Let the 1st positive integer = x

2nd consecutive odd positive integer = $x+2$.

$$x^2 + (x+2)^2 = 130$$

$$= x^2 + x^2 + 4x + 4 = 130$$

$$= 2x^2 + 4x + 4 = 130. \text{ Make the equation } = 0.$$

$$= 2x^2 + 4x + 4 - 130 = 0$$

$$= 2x^2 + 4x - 126 = 0 \quad \text{Divide by 2.}$$

$$= x^2 + 2x - 63 = 0$$

Factor it.

$$(x+9)(x-7) = 0$$

$$x = -9 \quad x = 7 \checkmark$$

Integers are 7 & 9.

14. Factor the trinomial:

$$r^2 + 6rg - 27g^2$$

Solution -

$$\text{Product} = -27 \quad ; \quad \text{Sum} = +6$$

The numbers are +9, -3

$$(r+9g)(r-3g) \checkmark$$

15. Factor: $7s^2 - 49s + 84$

- Solution -

Take "7" as a common factor.

$$7(s^2 - 7s + 12) \quad \text{Product} = 12, \text{Sum} = -7$$

$$7(s-4)(s-3) \checkmark$$

16. Factor by grouping: $21x^3 - 15x^2 + 56x - 40$

Solution -

$$(21x^3 - 15x^2) + (56x - 40)$$

Take a common factor of each group.

$$3x^2(7x-5) + 8(7x-5).$$

Take $(7x-5)$ as a common factor.

$$(7x-5)(3x^2+8) \checkmark$$

17 Factor the trinomial completely

$$6w^2 - 28w + 16$$

- Solution -

$$6w^2 - 28w + 16 \quad \text{Take } 2 \text{ as a common factor.}$$

$$2(3w^2 - 14w + 8)$$

Now, factor $3w^2 - 14w + 8$.

Multiply 3 by 8 = 24 \therefore Product = 24, Sum = -14
numbers are -12 & -2

$$= (3w - 12)(3w - 2) \quad \text{Divide the } 1^{\text{st}} \text{ term by 3.}$$

$$= (w - 4)(3w - 2)$$

Answer is : $2(w - 4)(3w - 2)$ ✓

18 Factor: $6x^6y^4 + 48x^4y^3 + 18xy$

Solution -

$$6x^6y^4 + 48x^4y^3 + 18xy$$

$6xy$ is the common factor.

$$6xy(x^5y^3 + 8x^3y^2 + 3) \checkmark$$

19 Solve the equation:

$$3b^2 - 10b = 8$$

- Solution -

Make the equation = 0

$$3b^2 - 10b - 8 = 0 \quad \text{factor it:}$$

$$3(-8) = -24 \rightarrow \text{product} = -24, \text{sum} = -10$$

Numbers are -12 and +2

$$= (3b - 12)(3b + 2). \quad \text{Divide the } 1^{\text{st}} \text{ term by 3.}$$

$$= (b - 4)(3b + 2) = 0 \quad \text{Set each factor to 0.}$$

$$b - 4 = 0 \rightarrow b = 4; \quad 3b + 2 = 0 \rightarrow b = -\frac{2}{3} \checkmark$$

20 Factor the Polynomial twice:

$$-2x^5 + 8x^4 - 10x$$

- Solution -

Take -1 as a common factor.

$$-1(2x^5 - 8x^4 + 10x)$$

Now factor $2x^5 - 8x^4 + 10x$.

The common factor is $2x$

$$2x(x^4 - 4x^3 + 5)$$

Answer is: $-2x(x^4 - 4x^3 + 5) \checkmark$

21 Factor:

$$3(x+7)^2 + 20(x+7) + 25$$

- Solution -

$$\text{Let } x+7 = u \Rightarrow 3u^2 + 20u + 25.$$

$$\text{Product} = 3(25) = 75, \text{ Sum} = 20.$$

Numbers are: +15, +5

$$(3u+15)(3u+5). \text{ Divide the } 1^{\text{st}} \text{ term by 3}$$

$$(u+5)(3u+5). \text{ Replace } u \text{ with } x+7.$$

$$(x+7+5)(3(x+7)+5) = (x+12)(3x+21+5)$$
$$= (x+12)(3x+26) \checkmark$$

22 Solve: $c(9c+2) = 7$

- Solution -

$$c(9c+2) = 7$$

$$\text{Distribute: } 9c^2 + 2c = 7.$$

Make the equation = 0.

$$9c^2 + 2c - 7 = 0$$

$$\text{Product} = 9(-7) = -63; \text{ Sum} = +2.$$

Numbers are: +9, -7

$$(9c+9)(9c-7) = 0. \text{ Divide the } 1^{\text{st}} \text{ term by 9.}$$

$$(c+1)(9c-7) = 0. \text{ Set each factor to 0.}$$

$$c+1 = 0 \Rightarrow c = -1 \checkmark$$

$$\frac{9c-7}{+7} = 0$$

$$9c = 7 \Rightarrow c = 7/9 \checkmark$$

23

Solve the equation:

$$(4x+7)(x-3) = 12x+21$$

Solution -

$$(4x+7)(x-3) = 12x+21$$

$$= 4x^2 + 7x - 12x - 21 = 12x + 21.$$

Make it = 0.

$$4x^2 + 7x - 12x - 21 - 12x - 21 = 0$$

Combine like terms.

$$4x^2 - 17x - 42 = 0 \quad \text{Factor it:}$$

$$(4x+7)(x-6) = 0 \rightarrow \text{Set each factor to 0}$$

$$4x+7=0 \rightarrow x = -7/4, x-6=0 \rightarrow x = 6$$

24,

$$\text{Factor: } 400b^2 - 361$$

- Solution -

$400b^2 - 361$; it is a difference of 2 squares

$$\sqrt{400b^2} = 20b, \sqrt{361} = 19$$

$$\text{Answer is } (20b+19)(20b-19)$$

25,

$$\text{Factor: } 125 - 216w^3$$

- Solution -

$$125 - 216w^3 \quad (\text{Difference of 2 cubes})$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$125 - 216w^3 = (5)^3 - (6w)^3 \text{ if } a=5, b=6w.$$

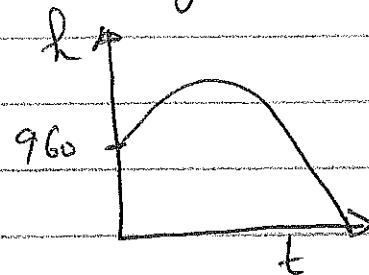
Replace $a=5 + b=6w$ in the formula;

$$(5 - 6w)(25 + 30w + 36w^2)$$

26, $h = -16t^2 + 64t + 960$

a, find the number of seconds until it returns to the ground.

b, find the number of seconds until the projectile is 880 ft above the ground.



Solution-

a, when it returns to the ground, $h=0$.

$$0 = -16t^2 + 64t + 960$$

Divide by -16

$$0 = t^2 - 4t - 60 \quad \text{Factor.}$$

$$0 = (t-10)(t+6) = 0 \Rightarrow t = 10 \text{ sec.}$$

b, $h = 880$

$$880 = -16t^2 + 64t + 960. \quad \text{Make it } = 0$$

$$-16t^2 + 64t + 960 - 880 = 0 \quad \text{Combine like terms.}$$

$$-16t^2 + 64t + 80 = 0 \quad \text{Divide by -16.}$$

$$t^2 - 4t - 5 = 0 \quad \text{Factor it.}$$

$$(t-5)(t+1) = 0 \Rightarrow t = 5 \checkmark$$

27

factors

$$(a+b)x^2 + (a+b)x - 20(a+b)$$

Solution -

Take $(a+b)$ as a common factor:

$$(a+b)(x^2 + x - 20)$$

Now factor $x^2 + x - 20$

Product = -20, Sum = +1

$$(a+b)(x+5)(x-4) \checkmark$$

28

find the greatest common factor.

$$110x^5, 70x^6, 60x^7$$

Solution

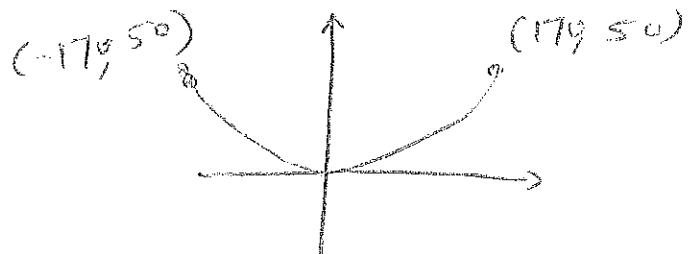
The greatest common factor of 110, 70, 60
is 10.

The greatest common factor of x^5, x^6, x^7 is x^5 .

Answer is $10x^5 \checkmark$

- Quadratic Functions -

1. A laboratory designed a radio telescope with a diameter of 340 feet and a maximum depth of 50 feet. The graph depicts a cross section of this telescope. Find the equation of this parabola.



- Solution -

$$y = ax^2$$

$$50 = a(17)^2 \rightarrow a = \frac{50}{17 \times 17}$$

dividing both sides by 5

$$= \frac{1}{34 \times 17} = \frac{1}{578}$$

$$y = \frac{1}{578} x^2$$

2. Let $g(x) = -x^2 + 5x + 2$ Find $g(\frac{1}{2})$

- Solution -

Replace x with $\frac{1}{2}$.

$$-\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) + 2$$

$$= -0.25 + 2.5 + 2 = 4.25 = 4\frac{1}{4}$$

$$= \frac{4 \times 4 + 1}{4} = \frac{17}{4} \checkmark$$

3 a) What is the maximum product whose

$$\text{sum} = -12$$

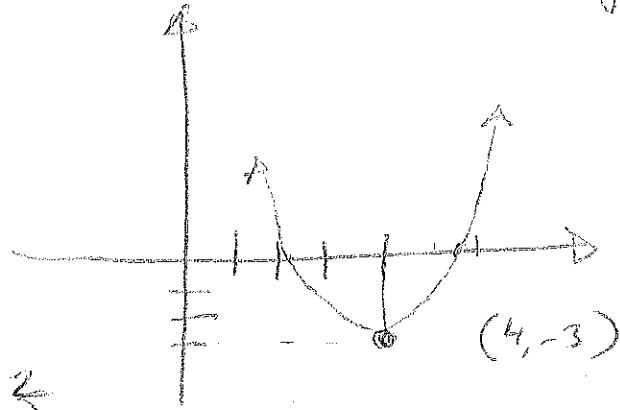
- Solution -

$$-12/2 = -6$$

$$(-6)(-6) = 36$$

b. What numbers yield this product?
-6, -6.

4. The graph of the Quadratic function is



$$f(x) = (x-4)^2 - 3$$

5. Sales of SUV for the year 1990 - 1999
(can be modeled by:

$$f(x) = .016x^2 + .124x + .787$$

$x=0$ represents 1990.

Sales

Use the model to approximate the
sales in 1994

Solution -

$$x = 0 \Rightarrow 1990$$

$$1994 \Rightarrow x = 4$$

Replace $x = 4$ in

$$+ 0.16x^2 + .124x + .787$$

$$= 0.16(4)^2 + .124(4) + .787$$

$$= 0.16 \times 16 + .496 + .787 =$$

$$+ 2.56 + .496 + .787 = 3.539$$

rounded to nearest 10th = 3.5

6. For the following quadratic function, tell whether the graph opens upward or downward and whether the graph is wider, narrower, or the same as $f(x) = x^2$

$$f(x) = -0.5x^2$$

Solution -

a) downward

b) wider

7. In the following exercise, find the coordinates of the vertex for the parabola

$$f(x) = -x^2 + 10x + 7$$

Solution -

$$a = -1, b = 10, c = 7.$$

$$\text{Vertex} = \frac{-b}{2a} = \frac{-10}{2(-1)} = \frac{-10}{-2} = 5$$

$$Y_{\text{vertex}} = -(5)^2 + 10(5) + 7$$

$$= -25 + 50 + 7 = 32.$$

Vertex $(5, 32)$.

8. Let $f(x) = x^2 - x + 3$
 $g(x) = 8x - 2$

Find: a) $g(3)$
— Solution —

Replace x with 3 in $g(x) = 8x - 2$

$$8(3) - 2 = 24 - 2 = 22 \checkmark$$

b) $f(g(3))$
— Solution —

Replace x with 22 in

$$\begin{aligned}f(x) &= x^2 - x + 3 \\&\text{with } = (22)^2 - 22 + 3\end{aligned}$$

$$= 484 - 22 + 3 = 465 \checkmark$$

12, If a baseball is projected upward from ground level with an initial velocity of 32 ft/sec, then its height is a function of time, given by $S = -16t^2 + 32t$

What is the maximum height reached by the ball?

- Solution -

$$S = -16t^2 + 32t$$

Maximum height is the vertex of the parabola.

$$a = -16, b = 32$$

$$x_{\text{vertex}} = t = \frac{-b}{2a} = \frac{-32}{2(-16)} = \frac{-32}{-32} = 1$$

Replace $t = 1$ in S

$$S = -16(1)^2 + 32(1)$$

$$= -16 + 32 = 16 \text{ ft}$$

13, The highest or lowest point of a parabola is called:

- Solution -

It is called the Vertex

9

Functions of the form $f(x) = ax^2 + bx + c$
are called _____

- Solution -

They are called Quadratic functions

- 10, Use the equation Price = $0.01125x^2 + 0.1575x + 1.50$
to find the price of a 14-inch pizza, where
 x is the pizza size.

Find the price of the 14-in pizza.

- Solution -

Replace x with 14 in the formula.

$$\begin{aligned} & 0.01125 \times 14^2 + 0.1575 \times 14 + 1.50 \\ & = 0.01125 \times 196 + 2.205 + 1.50 \\ & = 2.205 + 2.205 + 1.50 = \$5.91\checkmark \end{aligned}$$

- 11, Given the function $f(x) = x^2 - x + 1$, find:
a, $f(2)$

- Solution -

Replace x with 2.

$$\begin{aligned} & = (2)^2 - 2 + 1 \\ & = 4 - 2 + 1 = 3\checkmark \end{aligned}$$

b, $f(-7)$

- Solution -

Replace x with -7 .

$$(-7)^2 - (-7) + 1 = 49 + 7 + 1 = 57\checkmark$$

c, $f(0)$

- Solution -

$$\text{Replace } x \text{ with } 0 \Rightarrow (0)^2 - 0 + 1 = 1\checkmark$$

- Using the Quadratic Formula -

↳ Use the quadratic formula to solve

$$5x^2 = 3 + 3x$$

Round to the nearest 10th

- Solution -

$5x^2 = 3 + 3x$. Make it in the form.

$$ax^2 + bx + c = 0$$

$$5x^2 - 3x - 3 = 0$$

$$\begin{matrix} \uparrow & \uparrow & \uparrow \\ a & b & c \end{matrix} \Rightarrow a = 5, b = -3, c = -3$$

find $b^2 - 4ac$ first.

$$\begin{aligned} & (-3)^2 - 4(5)(-3) \\ & = 9 - (-60) = 9 + 60 = 69. \end{aligned}$$

The Quadratic formula is:

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-3 \pm \sqrt{69}}{2 \times 5} = \frac{3 \pm \sqrt{69}}{10} \end{aligned}$$

But $\sqrt{69} = 8.306$ (by calculator)

$$x_1 = \frac{3 + 8.306}{10} = \frac{11.306}{10} = 1.1 \quad (\text{Rounded to nearest 10}^{\text{th}}).$$

$$x_2 = \frac{3 - 8.306}{10} = \frac{-5.306}{10} = -0.5 \quad (\text{Rounded to nearest 10}^{\text{th}}).$$

2. Write the equation in standard form.

$$ax^2 + bx + c = 0$$

$$(x-6)(x+7) = 0$$

Find a , b , and c .

- Solution -

Multiply $(x-6)(x+7)$ first:

$$= x^2 - 6x + 7x - 42$$

$$= \begin{matrix} x^2 & + & x & - 42 \\ \uparrow & & \uparrow & \uparrow \\ a & b & c \end{matrix}$$

$$\Rightarrow a = 1, b = 1, c = -42$$

3. Use the Quadratic formula to solve the equation

$$\frac{2}{3}x^2 - x + \frac{13}{9} = 0$$

- Solution -

Multiply $\frac{2}{3}x^2 - x + \frac{13}{9} = 0$ by the L.C.D
which is 9

$$9 \cdot \frac{2}{3}x^2 - 9 \cdot x + 9 \cdot \frac{13}{9} = 0$$

$$= \frac{18}{3}x^2 - 9x + 13 = 0$$

$$= 6x^2 - 9x + 13 = 0$$

find the discriminant $\Delta = b^2 - 4ac$ first.

$$a = 6, b = -9, c = 13$$

$$= (-9)^2 - 4(6)(13)$$

$$= 81 - 312 = -231$$

Since Δ is negative \Rightarrow No real solutions.
 $\sim (D)$

4. Use the quadratic formula to solve.

$$5x^2 - 2x + 13 = 18x + 1$$

Solution -

Make it = 0.

$$5x^2 - 2x + 13 - 18x - 1 = 0$$

$$5x^2 - 20x + 12 = 0$$

$$a = 5, \quad b = -20, \quad c = 12$$

$$\begin{aligned} b^2 - 4ac &= (-20)^2 - 4(5)(12) \\ &= 400 - 240 = 160 \end{aligned}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{20 \pm \sqrt{160}}{10} = \frac{20 \pm \sqrt{16 \times 10}}{10}$$

$$= \frac{20 \pm 4\sqrt{10}}{10}.$$

Divide each # by 2.

$$= \frac{10 \pm 2\sqrt{10}}{5}$$

3 A rule for estimating the number of board feet of lumber that can be cut from a log depends on the diameter of the log. Solve for d .

$$\left(\frac{d-4}{4}\right)^2 = 49$$

- Solution -

$$\left(\frac{d-4}{4}\right)^2 = 49$$

Square root both sides.

$$\frac{d-4}{4} = \pm 7 \quad \text{use } +7 \text{ first.}$$

$$\frac{d-4}{4} = 7 \quad \text{cross multiply.}$$

$$d-4 = 28 \rightarrow d = 32 \checkmark$$

use -7 .

$$\frac{d-4}{4} = -7 \quad \text{cross multiply.}$$

$$d-4 = -28 \rightarrow d = -24 \checkmark$$

Are both answers reasonable?

No, since "d" can not be negative.

6. Write the equation in standard form $a x^2 + b x + c = 0$.

$$4x^2 = -12x$$

- Solution -

Make it $= 0$.

$$4x^2 + 12x = 0$$

$$a = 4, \quad b = 12, \quad c = 0.$$

7. Use the quadratic formula to solve

$$0.5x^2 = 2x + 0.5$$

use radicals as answers.

- Solution -

Make it $= 0$.

$$0.5x^2 - 2x - 0.5 = 0$$

multiply by 2.

$$x^2 - 4x - 1 = 0$$

$$a = 1, \quad b = -4, \quad c = -1$$

$$b^2 - 4ac = (-4)^2 - 4(1)(-1) \\ = 16 + 4 = 20.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{20}}{2} = \frac{4 \pm \sqrt{4 \times 5}}{2}$$

$$= \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$

3. Use the quadratic formula to solve

$$9x^2 - 7x - 5 = 0 \quad (\text{radicals})$$

- Solution -

$$a = 9, \quad b = -7, \quad c = -5$$

$$\begin{aligned} b^2 - 4ac &= (-7)^2 - 4(9)(-5) \\ &= 49 + 180 = 229 \end{aligned}$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{7 \pm \sqrt{229}}{18} \end{aligned}$$

9 Solve by using the formula.

$$7p^2 = -33p - 20$$

- Solution -

Make it $\equiv 0$

$$7p^2 + 33p + 20 = 0$$

$$a = 7, \quad b = 33, \quad c = 20$$

$$\begin{aligned} b^2 - 4ac &= (33)^2 - 4(7)(20) \\ &= 529 \end{aligned}$$

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-33 \pm \sqrt{529}}{14}$$

$$p_1 = \frac{-33 + 23}{14} = \frac{-10}{14} = -\frac{5}{7} \quad p_2 = \frac{-33 - 23}{14} = -4$$

10. When the sum of 8 and twice a positive number is subtracted from the square of the number, 0 results.

Find the number

- Solution -

$$x^2 - (2x + 8) = 0 \quad (\text{Translated to algebra from the problem})$$

$$\begin{matrix} \uparrow & x^2 - 2x - 8 = 0 \\ a & b & c \end{matrix}$$

$$a = 1, b = -2, c = -8$$

Since the number is positive, use the following formula:

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{aligned} \text{Now } b^2 - 4ac &= (-2)^2 - 4(1)(-8) \\ &= 4 - (-32) = 4 + 32 \\ &= 36. \end{aligned}$$

$$x = \frac{-2 + \sqrt{36}}{2(1)} = \frac{2 + \sqrt{36}}{2}$$

$$\text{But } \sqrt{36} = 6 \Rightarrow$$

$$x = \frac{2+6}{2} = \frac{8}{2} = 4 \checkmark$$

ii) If $b^2 - 4ac > 0$, there is ? solutions

- Solution -

If $b^2 - 4ac > 0 \rightarrow$ 2 real solutions

If $b^2 - 4ac = 0 \rightarrow$ 1 real solution.

If $b^2 - 4ac < 0 \Rightarrow$ no real solutions.

12. If the discriminant of a quadratic equation
 $= 0$, then the equation has -

- Solution -

1 real solution

13. Which of the following is a quadratic
equation written in standard form?

- Solution -

It should be of the form.

$$ax^2 + bx + c = 0$$

Example: $8x^2 - 10x = 0$

14. The _____ states that if $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- Solution -

It is called the quadratic formula.

15. A frog is sitting on a stump 24 feet above the ground. It hops off the stump and lands on the ground 8 feet away.

During its leap, its height h is given by

$$\text{the equation } h = -0.5x^2 + x + 24$$

Where x is the distance from the base

of the stump, and h is the height in feet.

How far was the frog from the base of the stump when it was 18.38 feet above the ground?

- Solution -

Replace h with 18.38

$$18.38 = -0.5x^2 + x + 24$$

Subtract 18.38
from both sides

$$0 = -0.5x^2 + x + 5.62$$

Multiply by -1.

$$0 = 0.5x^2 - x - 5.62$$

$$a = 0.5, \quad b = -1, \quad c = -5.62$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{(-1)^2 - 4(0.5)(-5.62)}}{2(0.5)}$$

$$= \frac{1 \pm \sqrt{1 + 2 \times 5.62}}{1}$$

$$= 1 \pm \sqrt{1 + 11.24} = 1 \pm \sqrt{12.24}$$

$$\text{Now } \sqrt{12.24} = 3.498$$

$$= 1 \pm 3.498 \quad (\text{Take the positive answer only})$$

$$= 1 + 3.498$$

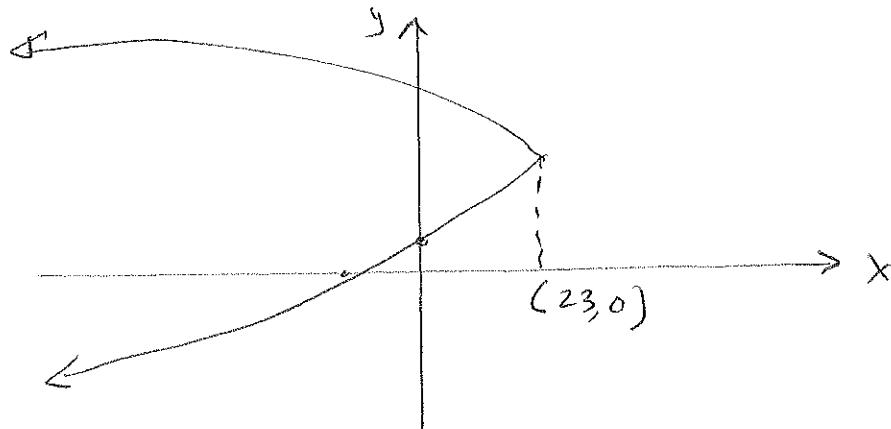
$$= 4.498 \quad \text{round to nearest 10th}$$

$$= 4.5 \checkmark$$

Graphs of Quadratic Functions

- b) Graph the parabola, and give the domain and range. $x = -(y - 5)^2 + 23$

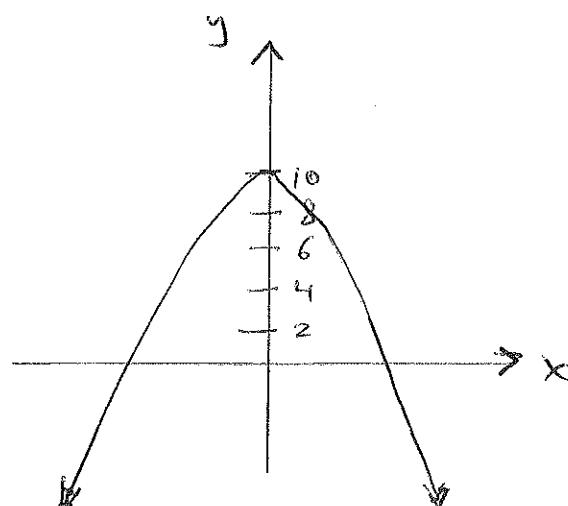
~ Solution ~



b, Domain: $(-\infty, 23]$

c, Range: $(-\infty, \infty)$

- 2, Find the domain and range.



~ Solution ~

Domain: $(-\infty, \infty)$

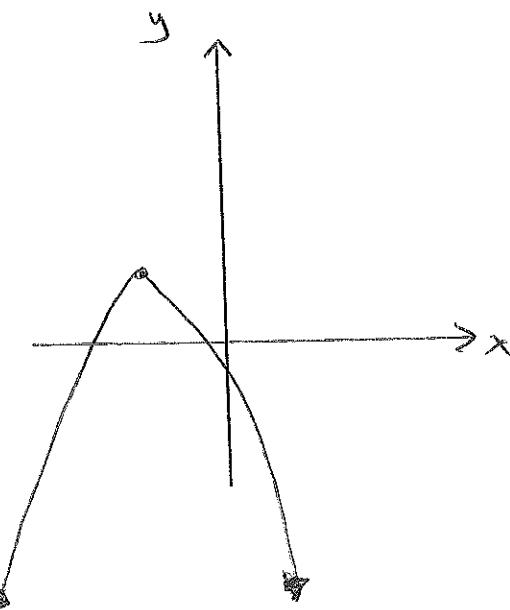
Range: $(-\infty, 10]$

3. Match the following equation with its graph.

$$y = -\frac{1}{2}x^2 - 3x - 4$$

- Solution -

Since the coefficient of x^2 is negative, the graph opens downward.



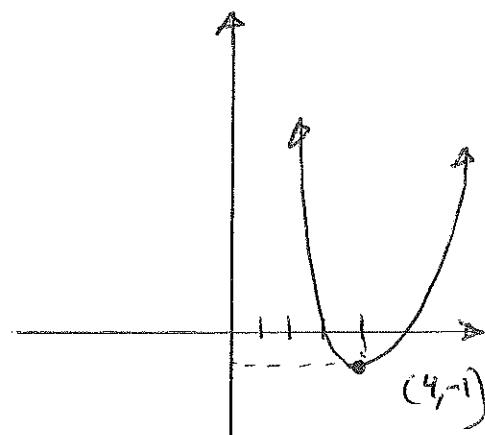
4. Use the graph of the quadratic function $f(x) = a(x-h)^2 + k$ to find the vertex, axis of symmetry and the minimum or maximum value of the function.

- Solution -

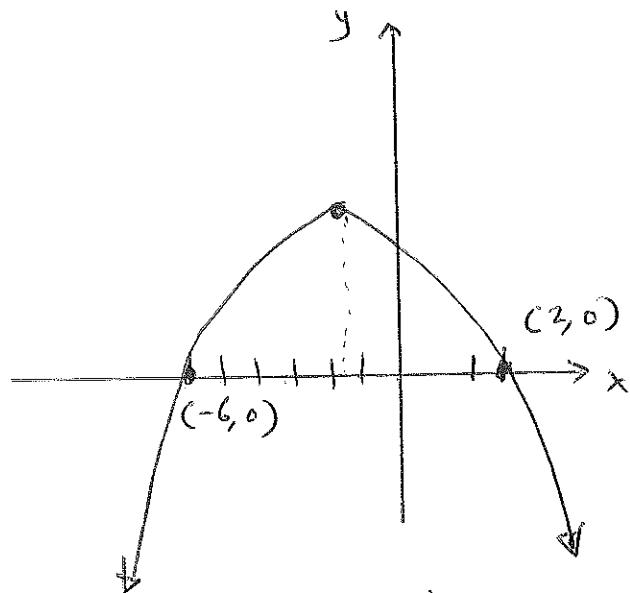
Vertex: $(4, -1)$

Range: $[-1, \infty)$

Minimum: -1



5 Find the x-intercepts of:



- Solution -

X-intercepts \Rightarrow where the graph crosses the x-axis.

If crosses the x-axis at:

$(3, 0)$ and $(-6, 0)$

6 Give the coordinates of the vertex and sketch the graph: $y = x^2 - 4$.

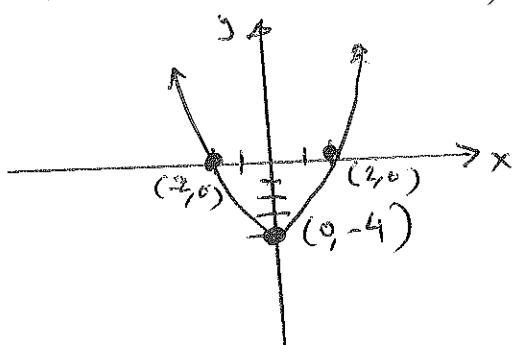
- Solution -

Coordinates of the vertex: $(0, -4)$.

For x-intercepts $\Rightarrow y = 0$

$$0 = x^2 - 4 \Rightarrow x^2 = 4$$

at $x = \pm 2$; therefore it crosses the x-axis at $(2, 0)$, $(-2, 0)$.



7) Graph the function and find the x-intercepts.

$$y = x^2 + 6x - 27$$

- Solution -

$$y = \begin{matrix} \uparrow x^2 \\ a \end{matrix} + \begin{matrix} \uparrow 6x \\ b \end{matrix} - \begin{matrix} \uparrow -27 \\ c \end{matrix}$$

$$a = 1, \quad b = 6$$

$$x\text{-value of the vertex} = \frac{-b}{2a} = \frac{-6}{2(1)} = \frac{-6}{2} = -3.$$

To find the y-value of the vertex,
replace $x = -3$ in $y = x^2 + 6x - 27$

$$\begin{aligned} y &= (-3)^2 + 6(-3) - 27 \\ &= 9 - 18 - 27 = -36 \end{aligned}$$

Vertex: $(-3, -36)$.

X-intercepts: Replace y with 0.

$$0 = x^2 + 6x - 27. \quad \text{Factor:}$$

$$0 = (x+9)(x-3)$$

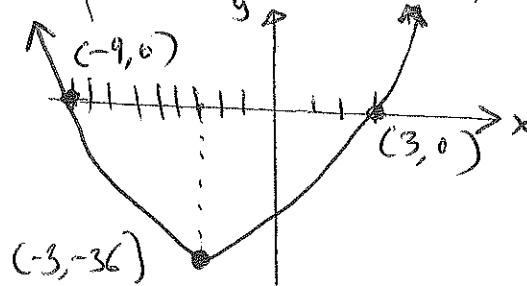
Set each factor to 0
and solve for x .

$$\begin{matrix} x+9 = 0 \\ -9 \quad -9 \end{matrix} \Rightarrow x = -9 \checkmark$$

$$\begin{matrix} x-3 = 0 \\ +3 \quad +3 \end{matrix} \Rightarrow x = 3 \checkmark$$

X-Intercepts are $-9, 3$

Graph is:



8) Find the Vertex, axis of symmetry, domain, and range for the parabola.

$$y = 2x^2 - 12x + 1$$

- Solution -

$$y = \underset{a}{\uparrow} 2x^2 - \underset{b}{\uparrow} 12x + 1$$

$$a = 2, b = -12$$

$$x_{\text{vertex}} = \frac{-b}{2a} = \frac{-12}{2(2)} = \frac{12}{4} = 3$$

To find the y-value of the vertex, replace $x = 3$ in:

$$\begin{aligned} y &= 2x^2 - 12x + 1 \\ y &= 2(3)^2 - 12(3) + 1 \\ &= 2(9) - 36 + 1 \\ &= 18 - 36 + 1 = -17 \end{aligned}$$

Vertex: $(3, -17)$.

Axis of symmetry: Is the x -value of the vertex.

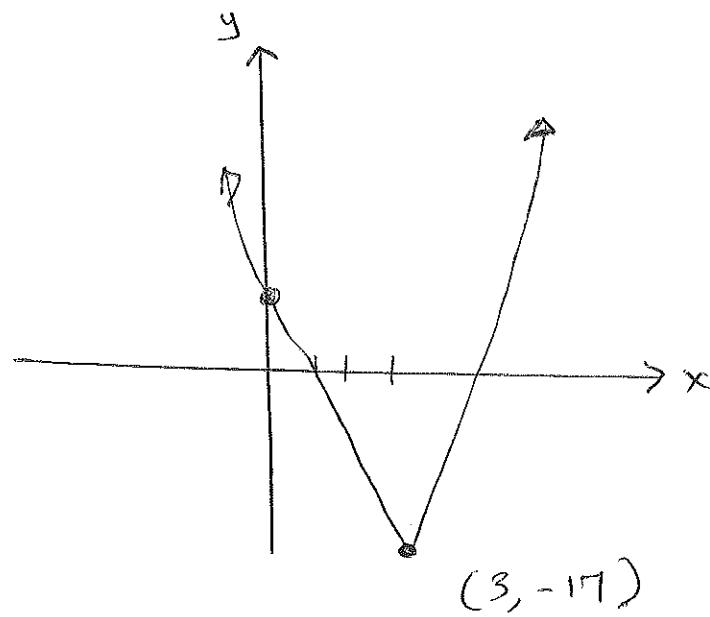
$$x = 3$$

Domain: $(-\infty, \infty)$.

Range: From the "y" value of the vertex to ∞

$$\text{Range: } [-17, \infty)$$

Graph :



9, Use the Vertex and y-intercepts to graph.

$$y - 1 = (x - 2)^2$$

- Solution -

$$y - 1 = (x - 2)^2$$

Solve for y by adding 1 to both sides of the equation.

$$y = (x - 2)^2 + 1$$

Vertex: $(2, 1)$.

Now find the y-intercept by replacing x with 0 in

$$y = (x - 2)^2 + 1$$

$$y = (0 - 2)^2 + 1 = 4 + 1 = 5.$$

Therefore y -intercept is $(0, 5)$.

Now plot the vertex $(2, 1)$ and the y -intercept $(0, 5)$ to graph the parabola.

