Study Guide to
3 major Pre-Calculus
Topics:

1. Graphs of rational functions.
2. Conic Section.
3. Graphs of Trigonometric functions
&
Finding the 6-Trig functions

Graphs of rational functions

Example 1 Graph: $y = \dfrac{2x+5}{x-1}$

Solution

Find the x-intercept first by setting $y = 0$.

$0 = \dfrac{2x+5}{x-1}$, a fraction $= 0$ when the numerator $= 0$ $\Rightarrow$ $2x + 5 = 0$ $\Rightarrow$ $2x = -5$; $x = -2.5$.

x-intercept is $(-2.5, 0)$

Find the y-intercept by setting $x = 0$.

$$y = \frac{2x+5}{x-1} = \frac{2(0)+5}{0-1} = -5$$

y-int $(0, -5)$.

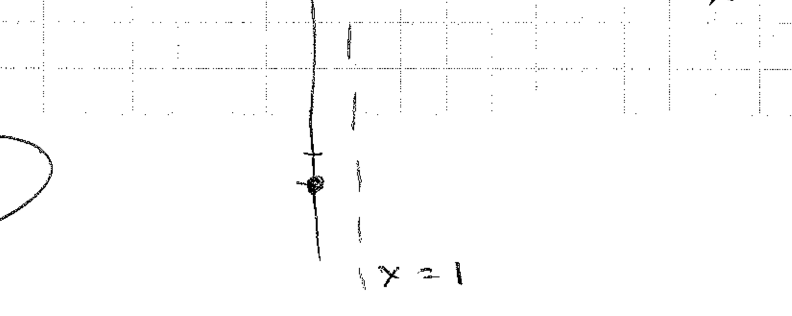
Find the vertical asymptote by setting the denominator to 0. $x - 1 = 0 \Rightarrow x = 1$.

Find the Horizontal asymptote. Since the numerator and denominator have the same degree (exponent $= 1$).

$$y = \frac{2}{1} = 2.$$

Plot all the calculated work:

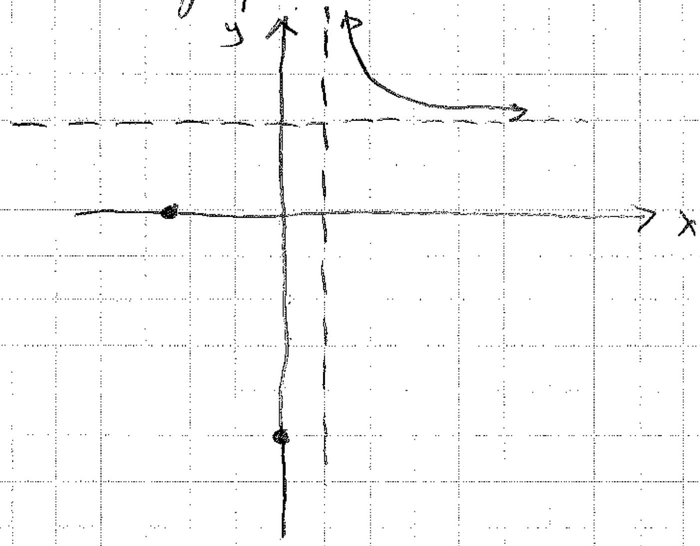
x-int $(-2.5, 0)$, y-int $(0, -5)$, V.A: $x = 1$, H.A: $y = 2$



Now pick a point on the right side of the Vertical asymptote which is any x larger than 1. let's pick $x = 2$. Substitute $x = 2$ in

$$y = \frac{2x+5}{x-1} = \frac{2(2)+5}{2-1} = \frac{9}{1} = 9$$

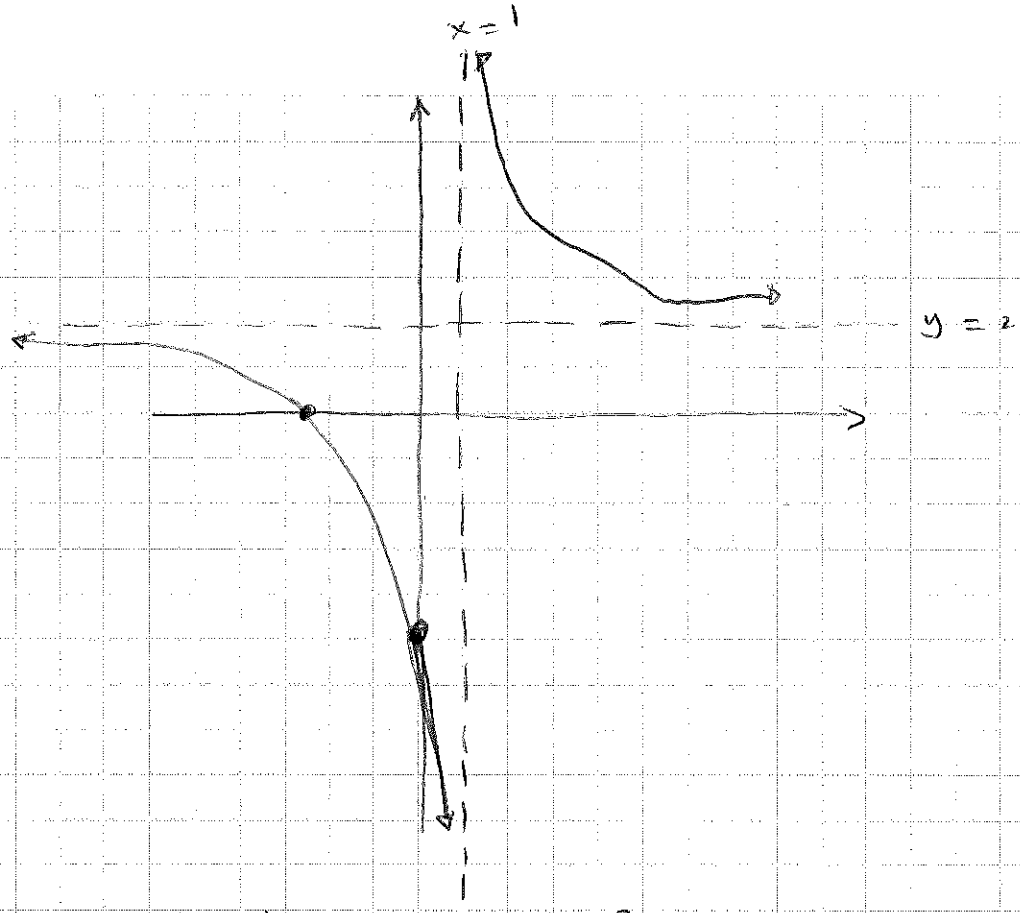
, since it is (+), the graph is on top of the x-axis:



Now pick a point on the left side of $x = 1$. let's pick $x = -1$, now find y

$$y = \frac{2(-1)+5}{-1-1} = \frac{-2+5}{-2} = \frac{3}{-2} = -1.5 .$$

Since y is (−), the graph is under the x-axis and it much pass through the intercepts



Example 2. Graph: $y = \dfrac{x+2}{x^2+1}$

– Solution –

x-intercept: Replace y with 0.

$$0 = \frac{x+2}{x^2+1} \implies x+2=0 \implies x=-2$$

$(-2, 0)$.

y-int: Replace x with 0.

$$y = \frac{0+2}{0^2+1} = 2 \implies (0, 2)$$

Vertical asymptote: Set the denominator to 0.

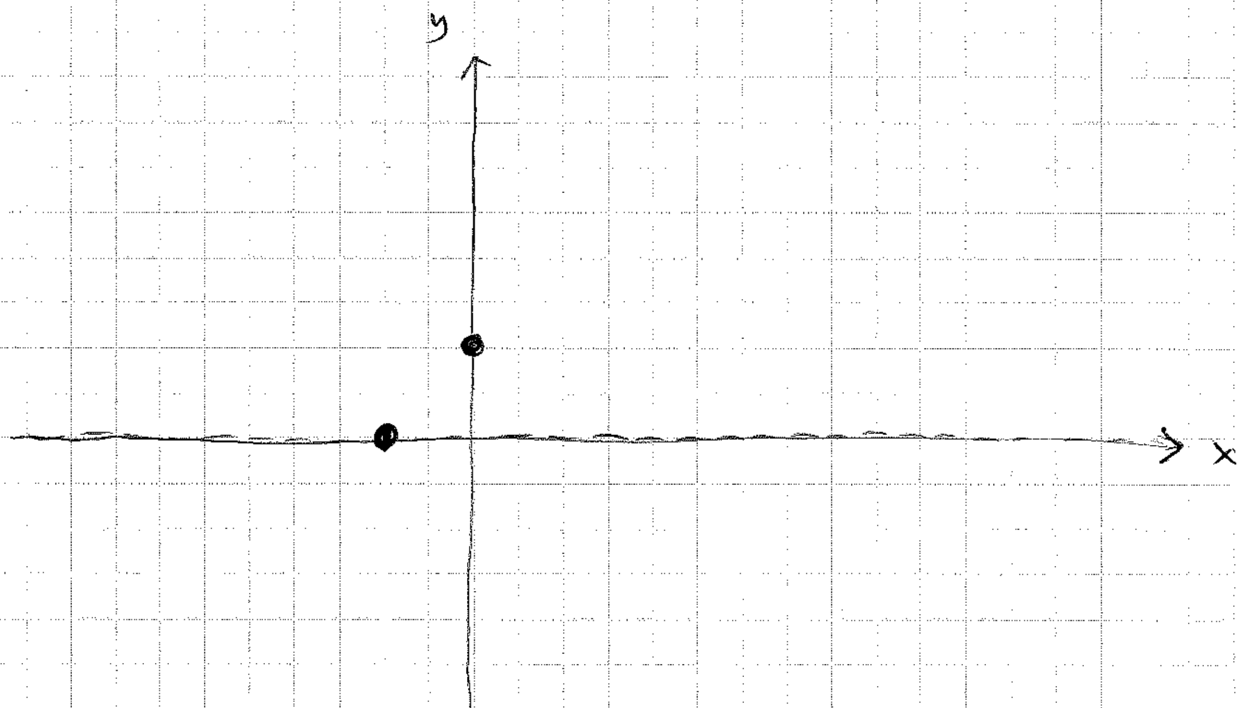
$$x^2+1=0 \implies \text{which is impossible}$$

since x^2 can not $= -1$.

(No Vertical asymptote)

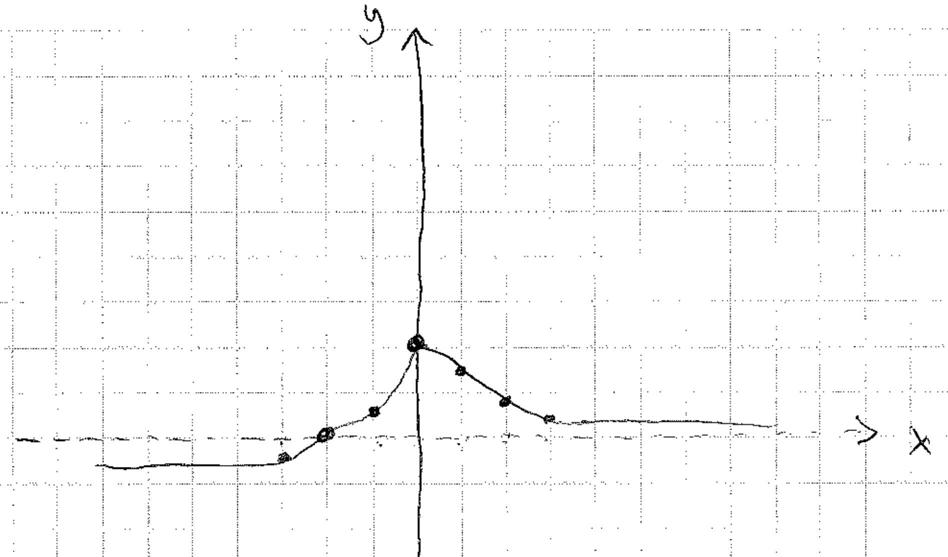
Horizonal asymptote: Since the degree or power of the denominator is larger than that of the numerator, the horizonal asymptote is $y=0$ which is the x-axis.

Plot all the calculated work:



Pick few points for x (some (−) and some (+)) and find y

x	−3	−2	−1	1	2	3
y	−0.1	0	0.5	1.5	0.8	0.5



Example 3: Graph: $y = \dfrac{x^2 - x - 2}{x - 2}$

– Solution –

x-int: $y = 0 \implies x^2 - x - 2 = 0$

Factor it: $(x-2)(x+1) = 0$

$x = 2$ and $x = -1$

$(2, 0)$, $(1, 0)$.

y-int: $x = 0 \implies y = \dfrac{(0)^2 - 0 - 2}{0 - 2} = 1$

$(0, 1)$

V. A. Set the denominator to 0.

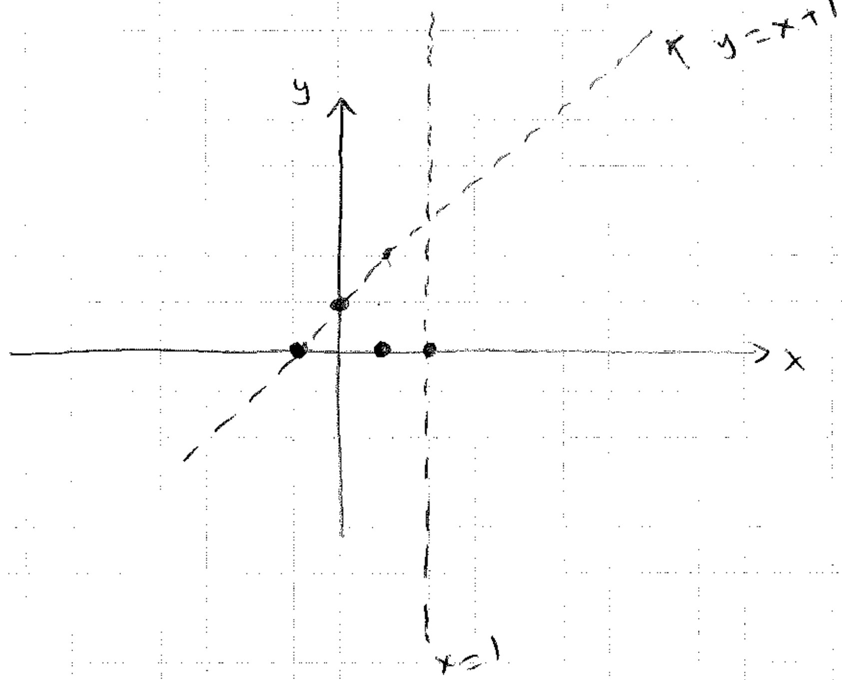
$x - 2 = 0 \implies x = 2$

H. A. Since the degree of the numerator is larger than the denominator, we will have a SLANT asymptote.

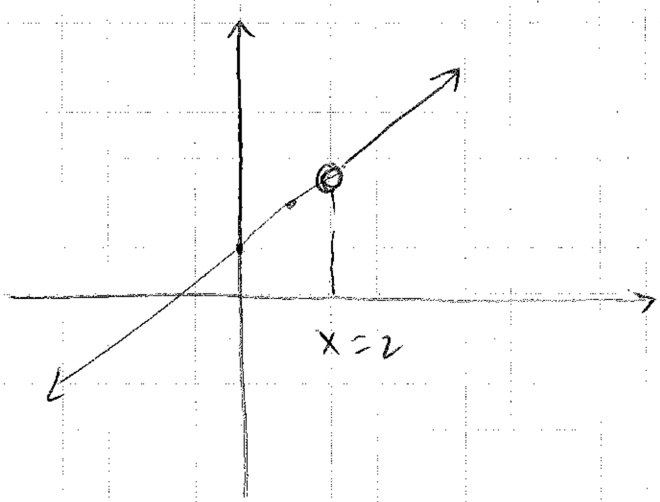
$y = \dfrac{x^2 - x - 2}{x - 2} = \dfrac{\cancel{(x-2)}(x+1)}{\cancel{(x-2)}}$

$y = x + 1$ is the SLANT ASYMPTOTE

Now plot all the calculated work:



The graph is actually a straight line $y = x+1$ with an open circle at $x = 2$



Example 4: Graph: $y = \dfrac{9}{x^2-9}$

x-int: $y = 0$

$0 = \dfrac{9}{x^2-9}$, but the numerator can not be 0.

Therefore, there is no x-intercept.

y-int: $x = 0$

$y = \dfrac{9}{0^2-9} = -1$ $(0, -1)$

V. A: Set the denominator to 0.

$x^2 - 9 = 0 \Longrightarrow x^2 = 9$

$x^2 = 9$

Take the $\sqrt{\ }$ of both sides of the equation.

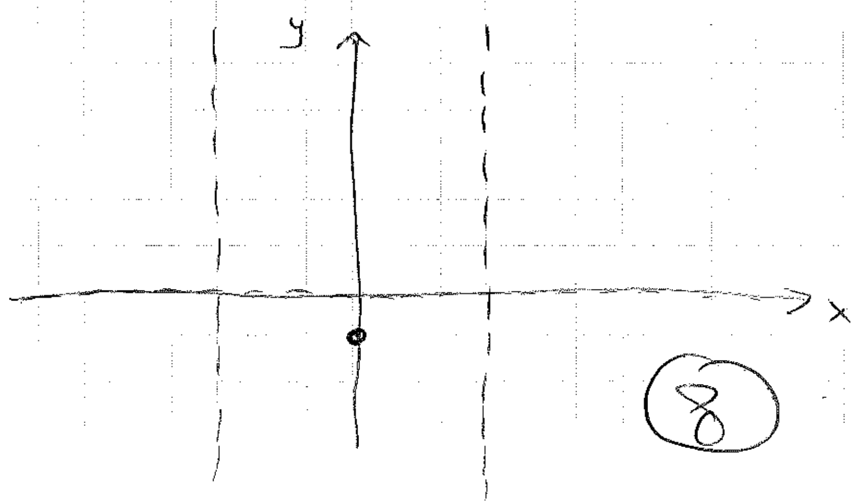
$\sqrt{x^2} = \sqrt{9} \Rightarrow x = \pm 3$

$x = 3$ and $x = -3$.

H. A. Since the numerator is constant.

$y = 0$. (x-axis)

plot all the calculated work:

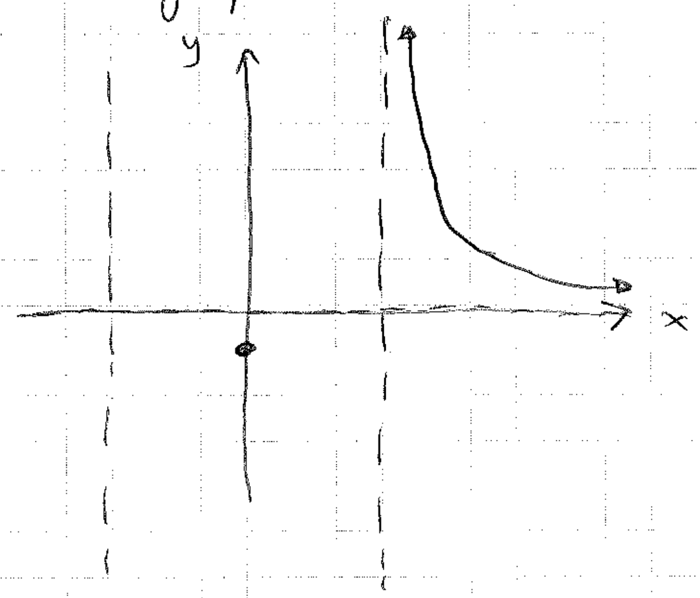


Now pick a point larger than $x=3$ (right side of the vertical asymptote).

Let's pick $x=4$. find y:

$$y = \frac{9}{(4)^2-9} = \frac{9}{16-9} = \frac{9}{7} \text{ (positive)}$$

which means the graph is on top of the x-axis.



Now pick a point between $x=0$ and $x=3$.

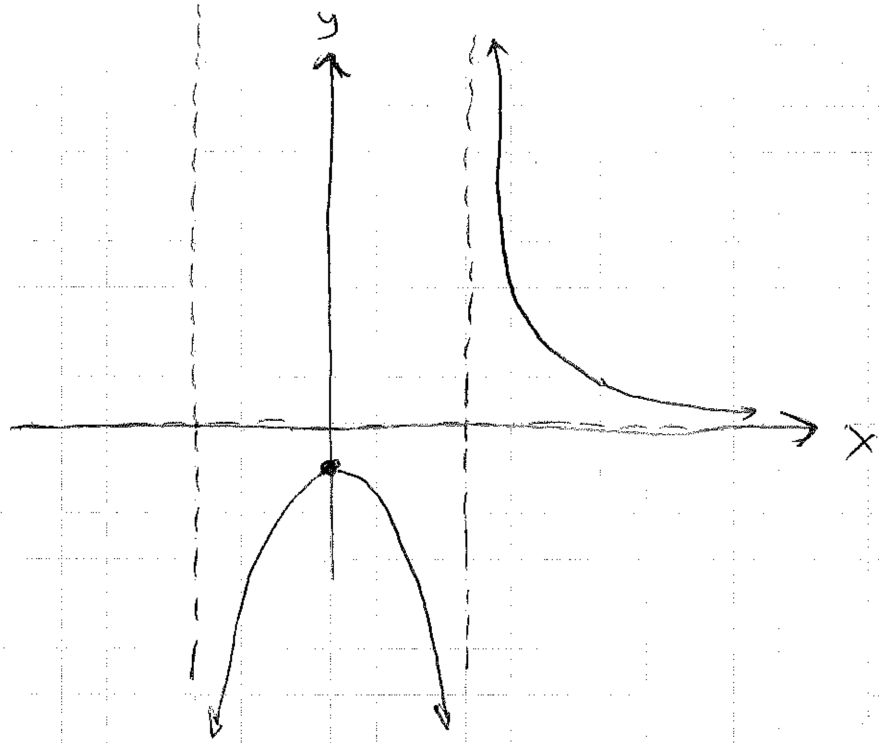
Let's pick $x=1$, find y:

$$y = \frac{9}{(1)^2-9} = \frac{9}{1-9} = \frac{9}{-8} \text{ (negative)}.$$

pick x between -3 and 0, let's pick $x=-1$

$$y = \frac{9}{(-1)^2-9} = \frac{9}{1-9} = \frac{9}{-8}, \text{ it's } (-)$$

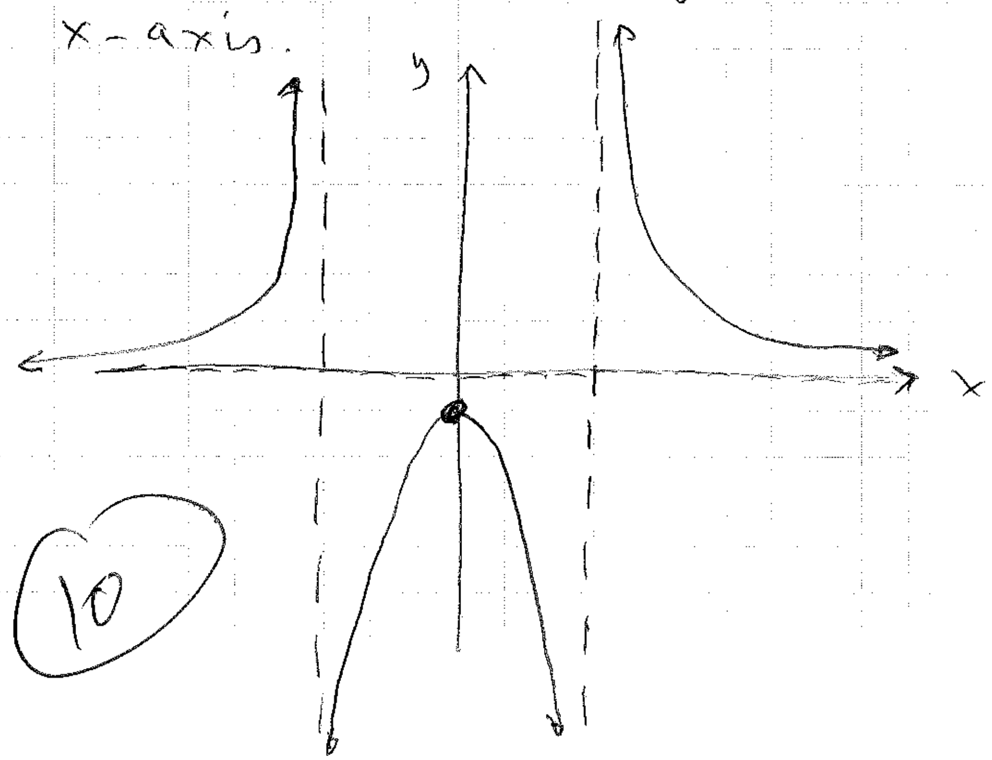
Therefore the graph between -3 and 3 is under the x-axis.



finally pick x to be smaller than −3, let's pick x = −4.

$$y = \frac{9}{(-4)^2 - 9} = \frac{9}{16-9} = \frac{9}{7}$$

. It's positive (+) which means the graph is on top of the x-axis.



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~ Ellipses ~

Example 1: Given the following equation.

$$9x^2 + 4y^2 = 36.$$

a, find the x and y-intercepts of the graph of the equation.

b, Find the coordinates of the foci.

c, Find the length of the major and minor axes

d, sketch the graph of the equation.

- Solution -

a, To find the x-intercept, set $y = 0$.

$$9x^2 + 4y^2 = 36$$

$$9x^2 + 4(0)^2 = 36 \Rightarrow 9x^2 = 36$$

$$x^2 = 4 \Rightarrow x = \pm 2 \qquad (-2, 0), (2, 0) \checkmark$$

y-intercepts: Set $x = 0$

$$9x^2 + 4y^2 = 36$$

$$9(0)^2 + 4y^2 = 36 \Rightarrow 4y^2 = 36$$

$$y^2 = 9 \Rightarrow y = \pm 3 \Rightarrow (0, -3), (0, 3) \checkmark$$

b, To find the coordinates of the foci:
General equation of the ellipse is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$9x^2 + 4y^2 = 36 \qquad \text{Divide each term by 36.}$$

$$\frac{9x^2}{36} + \frac{4y^2}{36} = \frac{36}{36}$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

$a^2 = 4 \quad + b^2 = 9$

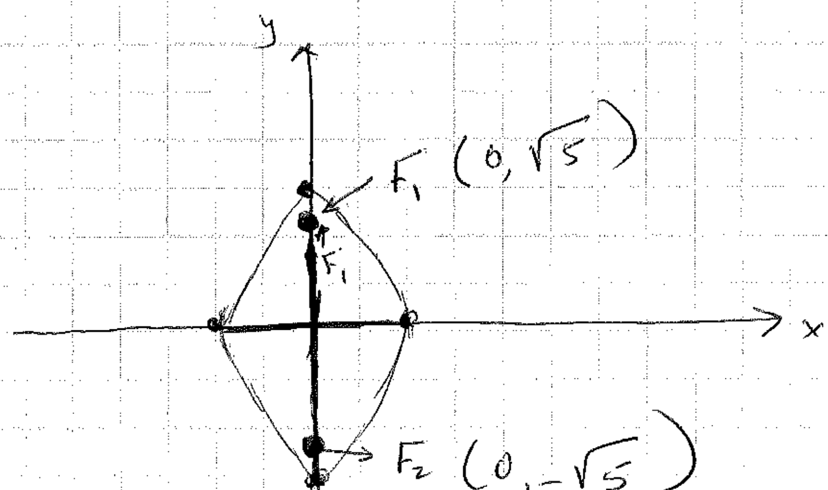
$c^2 = a^2 - b^2 = 9 - 4 = 5$

$c = \pm\sqrt{5} \quad f_1\ (0, -\sqrt{5}),\ (0, \sqrt{5})$

c, Major axis $= 2a = 2 \times 3 = 6$.

Minor axis $= 2b = 2 \times 2 = 4$

d,



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Example 2: Graph & find the foci of the following ellipse:

$$4x^2 + 9y^2 - 48x + 72y + 144 = 0$$

— Solution —

Combine the x-terms together and the y-terms together first:

$$(4x^2 - 48x) + (9y^2 + 72y) + 144 = 0$$

Take the coeficient of x^2 and y^2 as common factors:

$$4(x^2 - 12x) + 9(y^2 - 8y) + 144 = 0$$

Make each parenthesis as perfect square

$$4(x - \underset{\text{half of 12}}{6})^2 - 6 \times 6 \times 4 + 9(y - \underset{\text{half of 8}}{4})^2 - 4 \times 4 \times 9 + 144 = 0$$

$$4(x-6)^2 - 144 + 9(y-4)^2 - 144 + 144 = 0$$

$$4(x-6)^2 + 9(y-4)^2 = 144 \qquad \text{Divide by } 144$$

$$\frac{4(x-6)^2}{144} + \frac{9(y-4)^2}{144} = 1$$

$$= \frac{(x-6)^2}{36} + \frac{(y-4)^2}{16} = 1 \quad ✓$$

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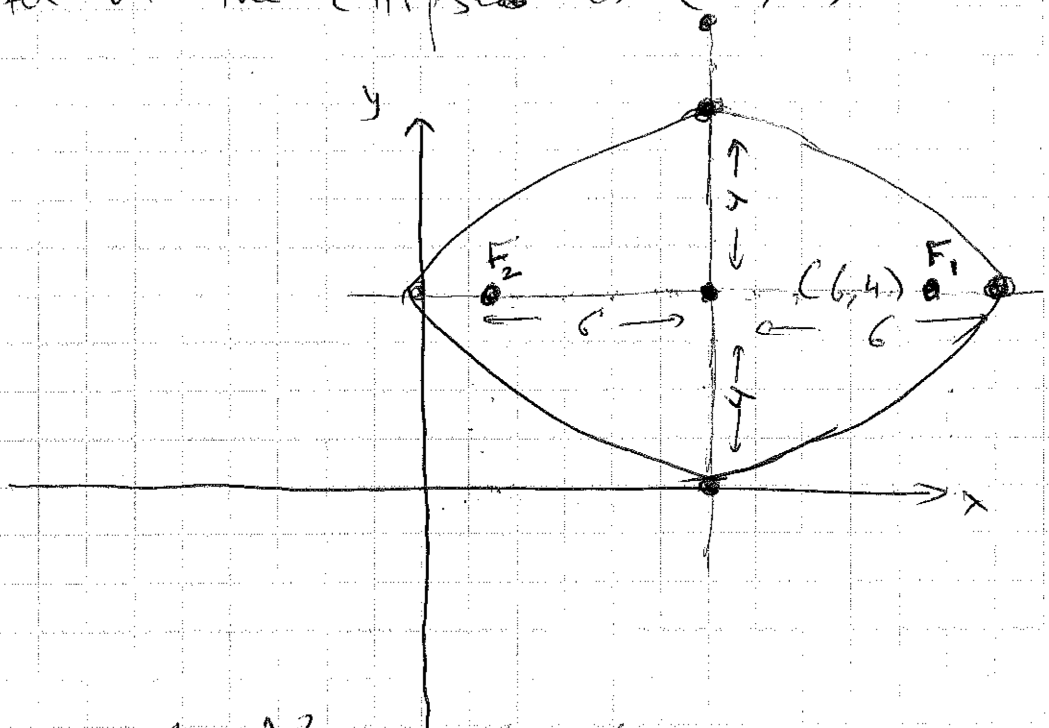
$a^2 = 36, \; b^2 = 16$

Center of the ellipse is (6, 4)



$c^2 = a^2 - b^2 = 36 - 16 = 20$

$c = \pm\sqrt{20} = \pm\sqrt{4 \times 5} = \pm 2\sqrt{5}$

or c (4.47, 0) , (-4.47, 0) from the new center

Foci from the old center :

$F_1\ (6 - 2\sqrt{5},\ 4)$, $F_2\ (6 + 2\sqrt{5},\ 4)$

— Hyperbolas —

Example 1: Find the center, vertices, foci, eccentricity, and asymptotes with the given equation, and sketch:

$$\frac{y^2}{25} - \frac{x^2}{144} = 1$$

— Solution —

$a^2 = 25$, $b^2 = 144$ $\Rightarrow a = 5$ & $b = 12$

c^2 (foci) $= a^2 + b^2 = 25 + 144 = 169 \Rightarrow c = \sqrt{169}$ $= 13$.

eccentricity $= \frac{c}{a} = 13/5$ ✓

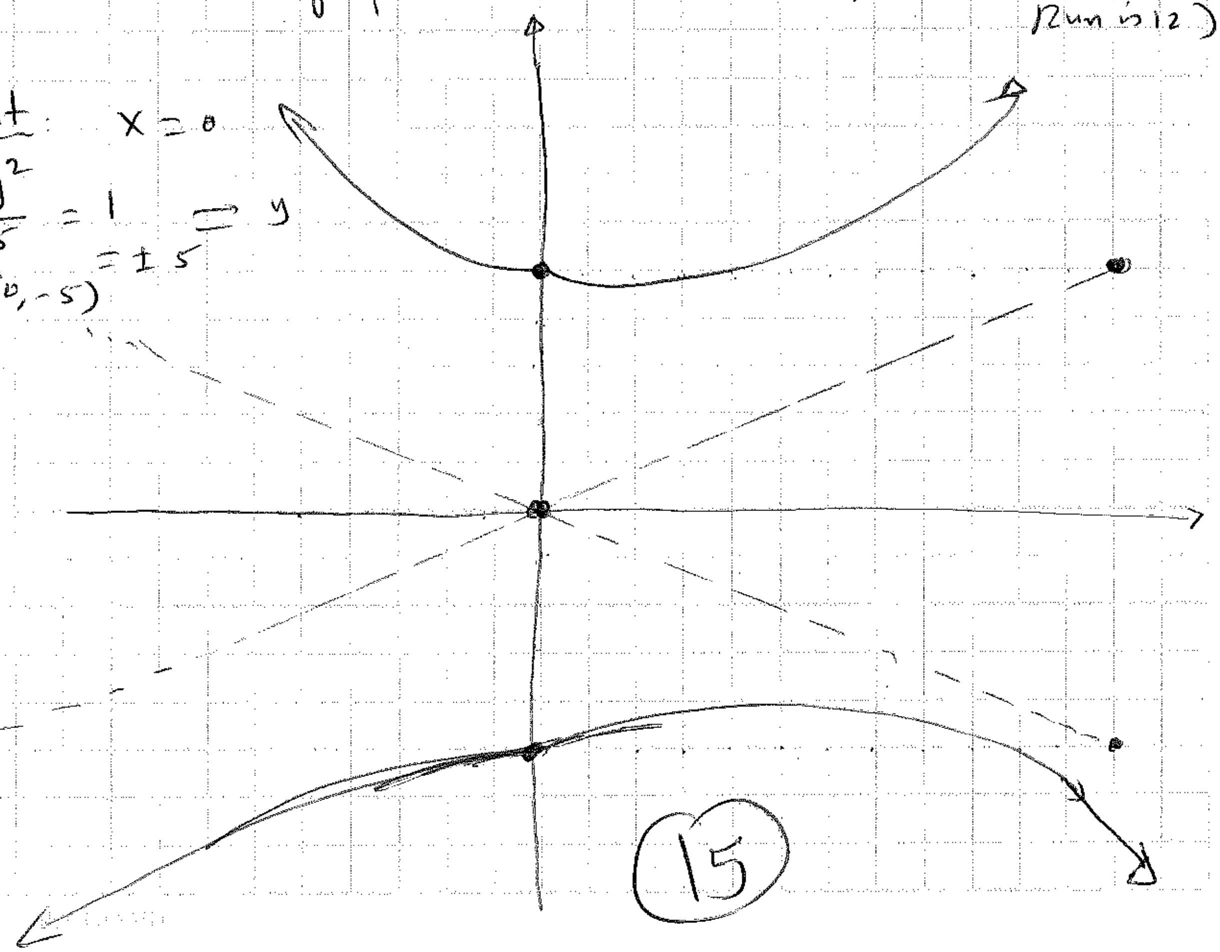
foci are on the y-axis (since it starts with y^2). Coordinates of foci $(0, 13)$ & $(0, -13)$

slope of the asymptote $= \pm a/b = \pm 5/12$ (Rise is 5, Run is 12)

<u>y-int</u>: $x = 0$

$\frac{y^2}{25} = 1 \Rightarrow y = \pm 5$

$(0, 5), (0, -5)$



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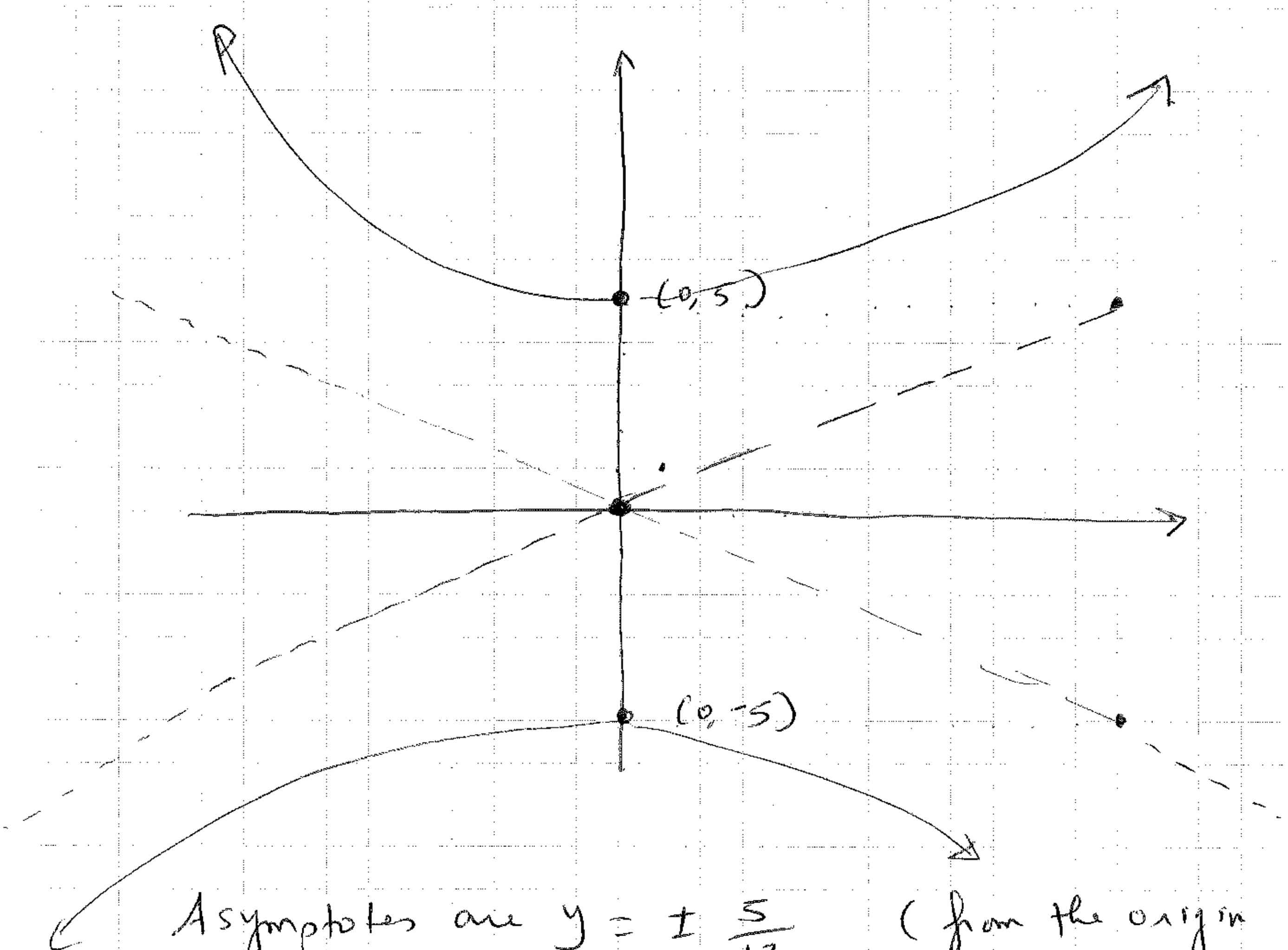
Here is a better graph:



Asymptotes are $y = \pm \frac{5}{12}$ (from the origin the rise is 5 and the run is 12

Center $(0, 0)$, Vertices are $(0, 5)$ and $(0, -5)$

Foci $(0, -13)$, $(0, 13)$

$e = 13/5$

Example 2: Find the center, vertices, and asymptotes of the hyperbola with equation:

$$4x^2 - 5y^2 + 40x - 30y - 45 = 0$$

Solution –

Combine the x terms together and the y terms together:

$$(4x^2 + 40x) - (5y^2 + 30y) - 45 = 0$$

Take "4" as a common factor for the x terms and "5" as common factor for the y terms.

$$4(x^2 + 10x) - 5(y^2 + 6y) - 45 = 0$$

Make perfect squares of each parenthesis:

$$4(\underbrace{x+5}_{\text{half of 10}})^2 - 5 \times 5 \times 4 - 5(\underbrace{y+3}_{\text{half of 6}})^2 + 3 \times 3 \times 5 - 45 = 0$$

$$4(x+5)^2 - 100 - 5(y+3)^2 + 45 - 45 = 0$$

$$4(x+5)^2 - 5(y+3)^2 = 100 \qquad \text{Divide by 100}$$

$$\frac{4(x+5)^2}{100} - \frac{5(y+3)^2}{100} = 1 \qquad \text{Reduce}$$

$$\frac{(x+5)^2}{25} - \frac{(y+3)^2}{20} = 1$$

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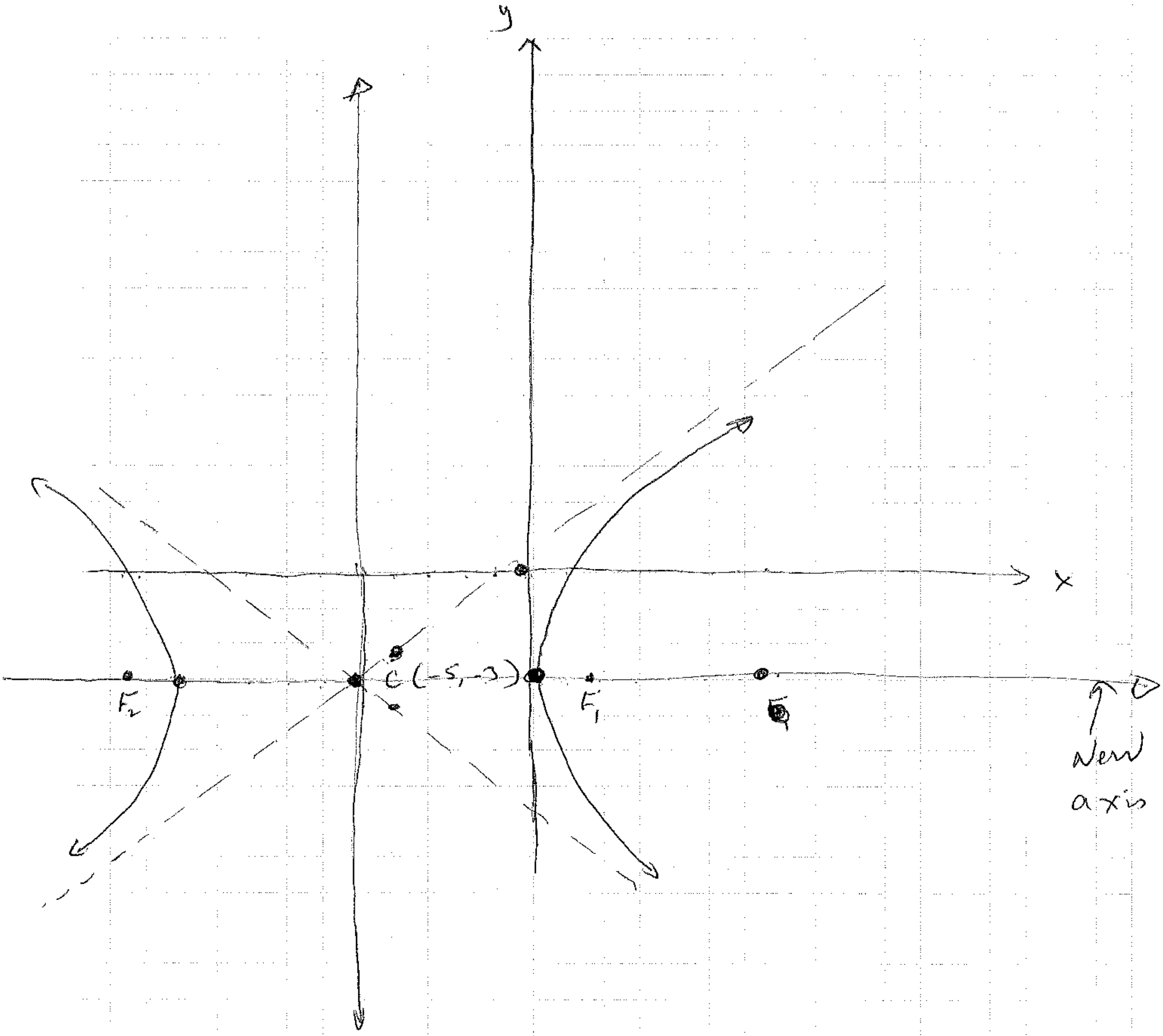
center is $(-5, -3)$ ✓

$a^2 = 25$, $b^2 = 20 \implies c^2 = 25 + 20 = 45$

$c = \pm\sqrt{45} = \pm\sqrt{9 \times 5} = \pm 3\sqrt{5}$

Since the equation starts with $x \Rightarrow$ foci are on the x-axis.



Equation of the asymptotes : $y = \pm \frac{b}{a} = \frac{\pm 2\sqrt{5}}{5} x$

or $y = \pm 0.894\, x$ from the new axis.

or $y = \pm \frac{2\sqrt{5}}{5}(x+5) - 3$

Foci $(-5-3\sqrt{5}, -3)$ + $(-5+3\sqrt{5}, -3)$

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– Circles –

<u>Example 1</u>: Find the center and radius of the circle of equation:

$$x^2 + y^2 + 2x + 8y + 8 = 0$$

– Solution –

Group the x terms together and the y terms together:

$$(x^2 + 2x) + (y^2 + 8y) + 8 = 0$$

Make the terms in parenthesis as perfect squares.

$$(x + \underset{\substack{\text{half}\\\text{of}\\2}}{1})^2 - 1 \times 1 + (y + \underset{\substack{\text{half}\\\text{of}\\8}}{4})^2 - 4 \times 4 + 8 = 0$$

$$(x + 1)^2 - 1 + (y + 4)^2 - 16 + 8 = 0$$

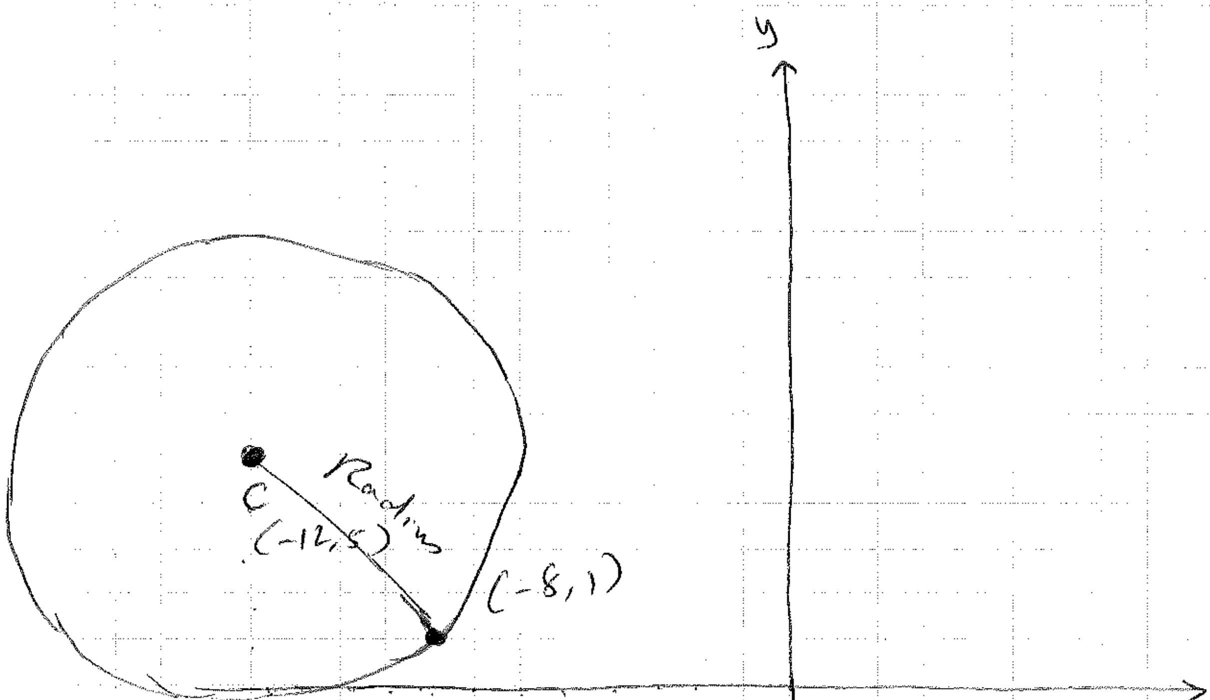
$$(x + 1)^2 + (y + 4)^2 = 25$$

center $(-1, -4)$, $r = \sqrt{25} = 5$ ✓

<u>Example 2</u>: What is the standard form of the equation of the circle with center $(-12, 5)$ and one point on the circle that has coordinates $(-8, 1)$.

– Solution –

Draw the circle.



The 2 pts we have (-12, 5), (-8, 1) create the radius. Find the length of the radius by using the distance formula:

$$(\underset{x_1}{-12}, \underset{y_1}{5}), \quad (\underset{x_2}{-8}, \underset{y_2}{1})$$

$$d = r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-8 - -12)^2 + (1-5)^2}$$

$$= \sqrt{4^2 + 4^2} = \sqrt{32}$$

or $r^2 = 32$. $\quad C(\underset{a}{-12}, \underset{b}{5})$.

Equation of the circle is:

$$(x - a)^2 + (y - b)^2 = R^2$$

$$(x + 12)^2 + (y - 5)^2 = 32$$

~ Parabola ~

Example 1: State the Vertex and focus of the parabola having the equation:

$$(y-3)^2 = 8(x-5)$$

– Solution –

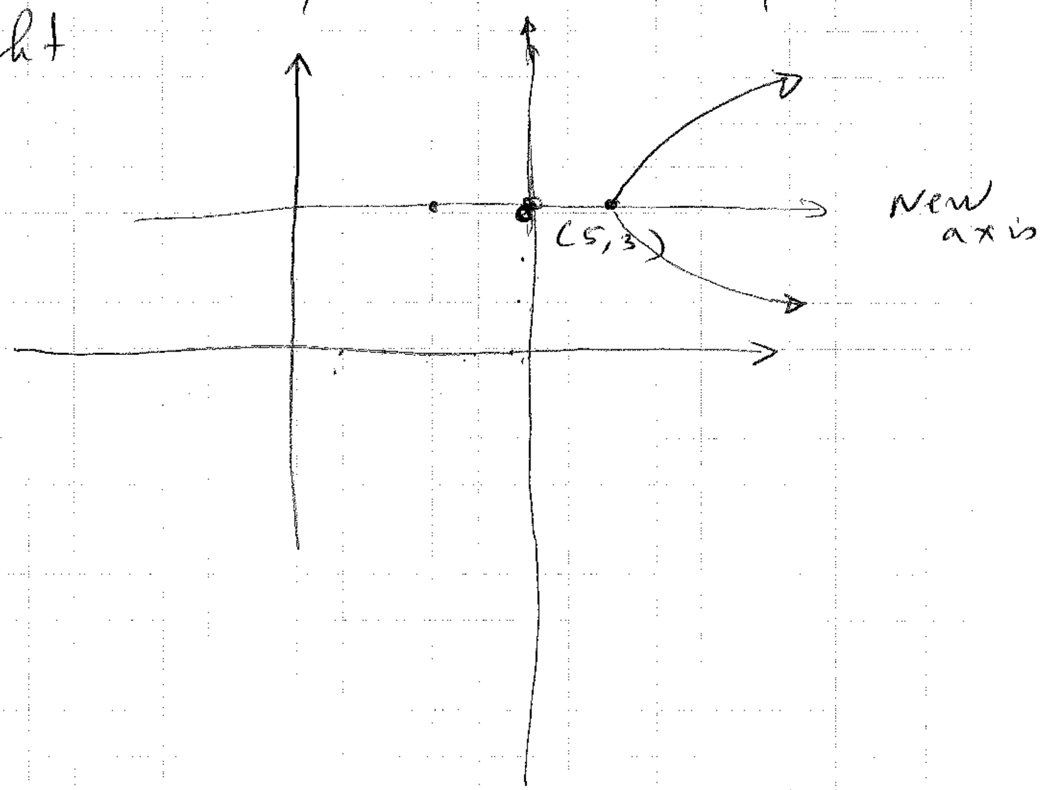
Vertex is $(5, 3)$

$4p$ for the unsquared part of the parabola is 8.

$4p = 8 \implies p = 2$.

Since y is the squared term, it opens to the right

Focus now is $(7, 3)$



Example 2: sketch $(x+4)^2 = -12(y+1)$

- Solution -

Vertex $(-4, -1)$

$4p = -12 \implies p = -3$ (focus)

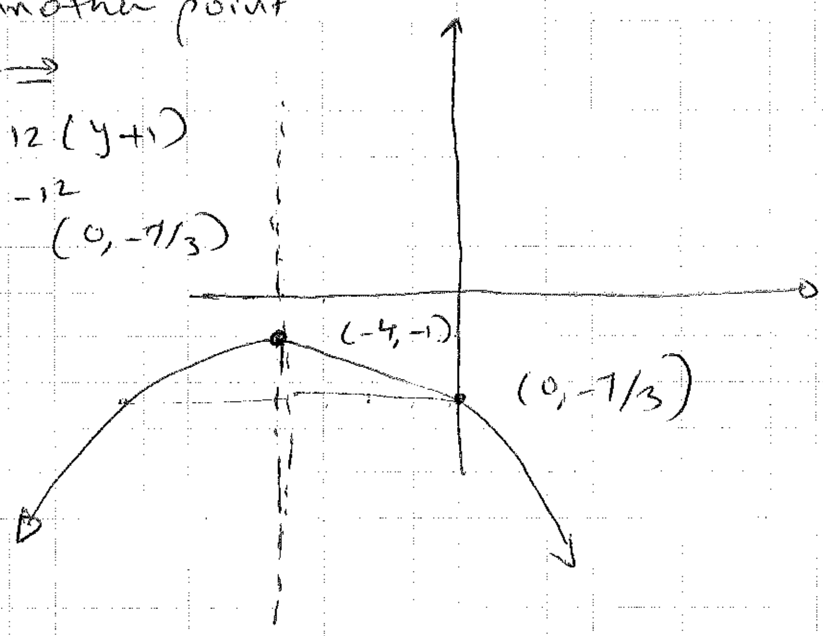
Pick another point

$x = 0 \implies$

$(4)^2 = -12(y+1)$

$16 = -12y - 12$

$y = -7/3$ $(0, -7/3)$



The 2^{nd} portion of the parabola is symmetric with respect to the vertex

Example 3: Sketch $y^2 - 2y + 4x - 12 = 0$

- Solution -

Group the y terms together first:

$(y^2 - 2y) + 4x - 12 = 0$

Make the group as a perfect square.

$(y-1)^2 - 1 \times 1 + 4x - 12 = 0$

half of 2

$(y-1)^2 - 1 + 4x - 12 = 0$

$(y-1)^2 = -4x + 13$

Take -4 as a common factor:

$(y-1)^2 = -4(x - 13/4)$

Therefore the coordinates of the vertex is: $(13/4, 1)$

Find the y-intercepts $\Rightarrow x = 0$.

$(y-1)^2 = -4(-13/4)$

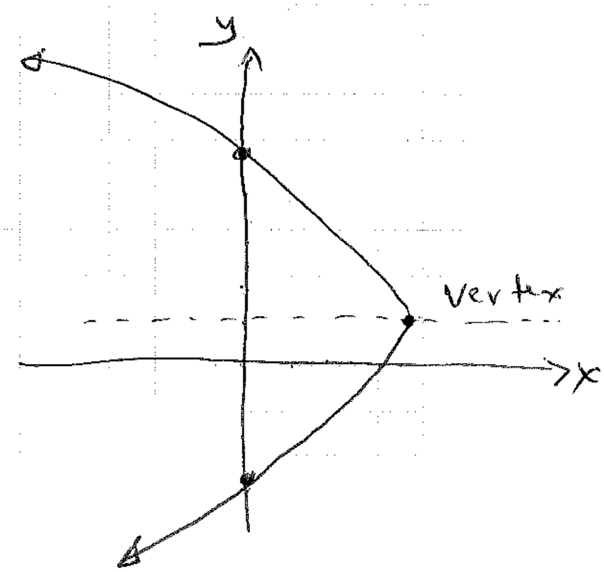
$(y-1)^2 = 13 \Longrightarrow y - 1 = \pm\sqrt{13}$ or $y - 1 = \pm 3.60$

so $y = 1 \pm 3.60$.

y-int: $(0, 4.60)$, $(0, -2.60)$

Directrix: $4p = -4$

$\Rightarrow p = -1$



— Graphs of Trigonometric Functions —

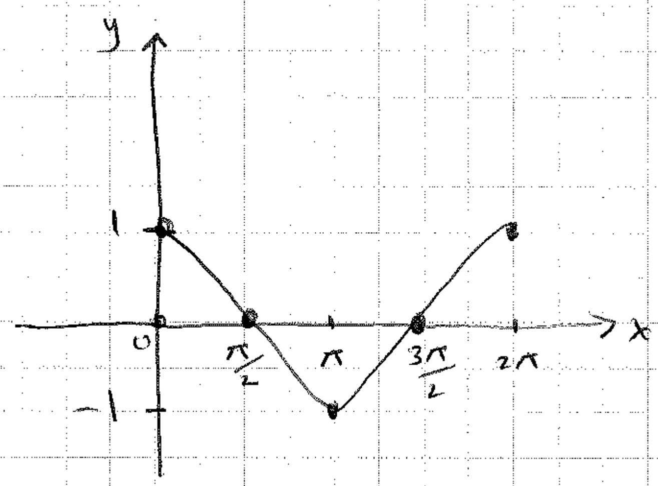
Graphs of $\cos x$:

Example 1: Graph: $y = \cos x$

— Solution —

Period is 2π.

x	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1



Example 2: Graph: $y = \cos \frac{1}{3}x$ or $\cos \frac{x}{3}$

— Solution —

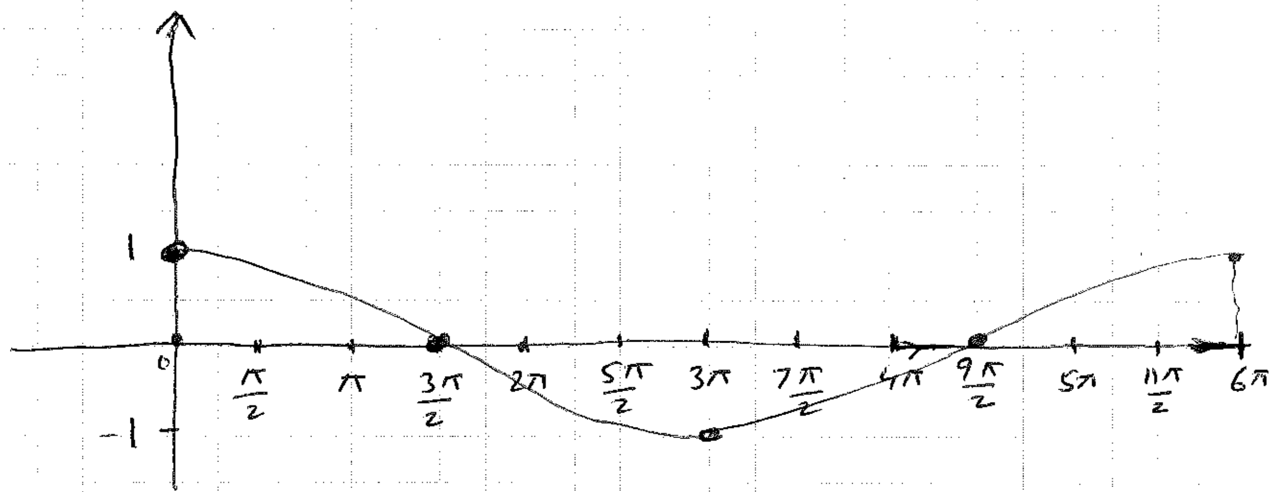
The period is $\dfrac{2\pi}{\frac{1}{3}} = 6\pi$.

multiply each angle by 3, but keep the y-value the same.

x	y
0	1
$3 \times \pi/2$	0
$3 \times \pi$	-1
$3 \times 3\frac{\pi}{2}$	0
$3 \times 2\pi$	1

or

x	y
0	1
$3\pi/2$	0
3π	-1
$9\pi/2$	0
6π	1



Example 3: Graph: $y = 3\cos 4x$

- Solution -

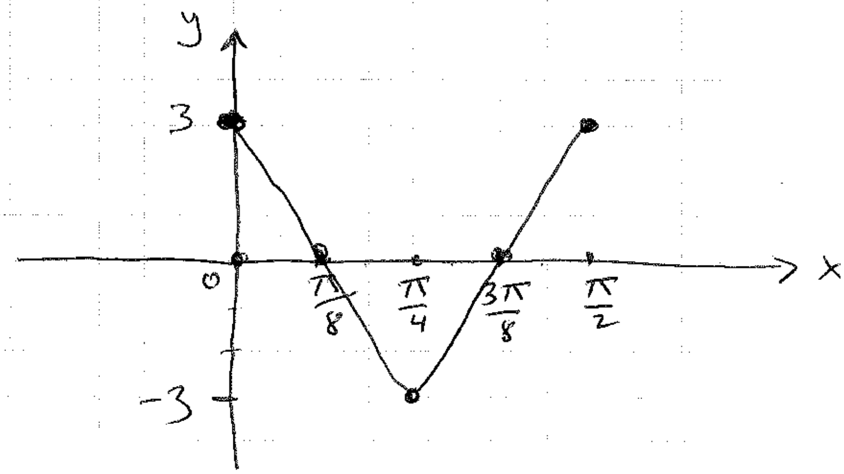
$$\text{Period} = \frac{2\pi}{4} = \pi/2$$

Divide each angle by 4 and multiply each y by 3.

x	New x	y	New y
0	0	1	3
$\frac{\pi}{2}$	$\pi/2 \div 4$	0	0
π	$\pi/4$	-1	-3
$3\pi/2$	$3\pi/2 \div 4$	0	0
2π	$2\pi \div 4$	1	3

New x	New y
0	3
$\pi/8$	0
$\pi/4$	-3
$3\pi/8$	0
$\pi/2$	3

All x values shrink by 1/4.



Example 4: Graph: $y = -2\cos(2x - \pi)$

– Solution –

$y = -2\cos(2x - \pi)$

Take 2 as a common factor.

$y = -2\cos 2\left(x - \frac{\pi}{2}\right)$

Period $= \frac{2\pi}{2} = \pi$.

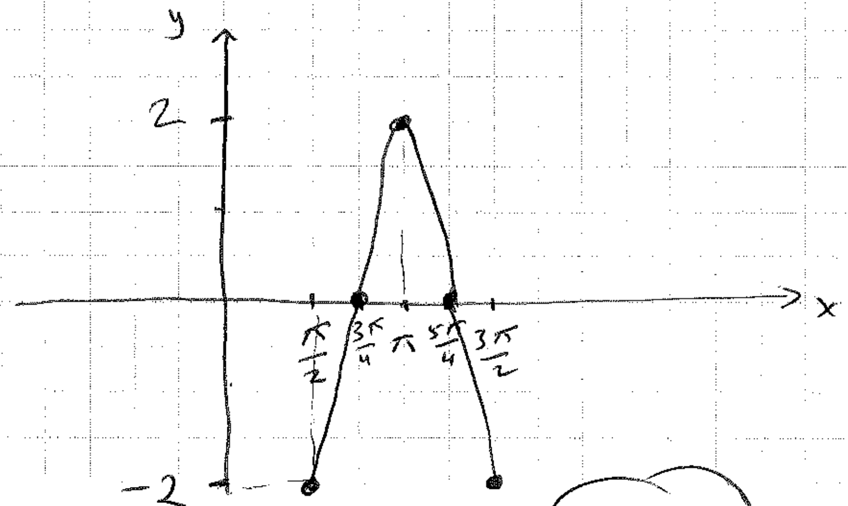
Divide each x value by 2 and since we have $-\frac{\pi}{2}$, add $\frac{\pi}{2}$ to each x-value.

Multiply each y value by -2.

x	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

New x	New y
$0/2 + \pi/2$	1×-2
$\pi/2 \div 2 + \pi/2$	0×-2
$\pi \div 2 + \pi/2$	-1×-2
$3\pi/2 \div 2 + \pi/2$	0×-2
$2\pi/2 + \pi/2$	1×-2

New x	New y
$\pi/2$	-2
$3\pi/4$	0
π	2
$5\pi/4$	0
$\frac{3\pi}{2}$	-2



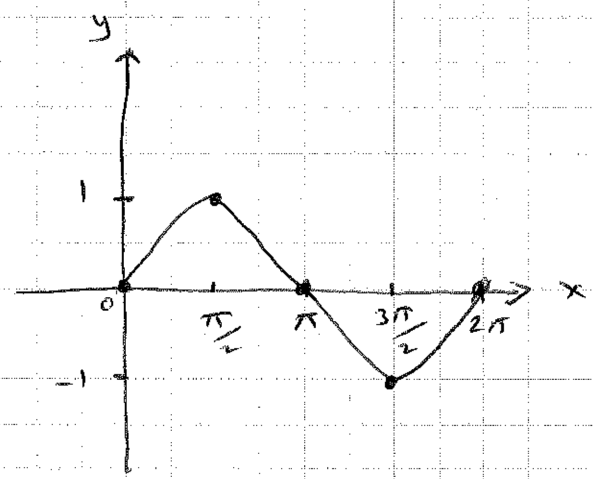
Graphs of Sin x

Example 1: Graph: $y = \sin x$

– Solution –

Period $= 2\pi$.

x	y
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0



Example 2.
Graph: $y = 3\sin(4x + \pi/2)$

– Solution –

$y = 3\sin(4x + \pi/2)$. Take 4 as a common factor.

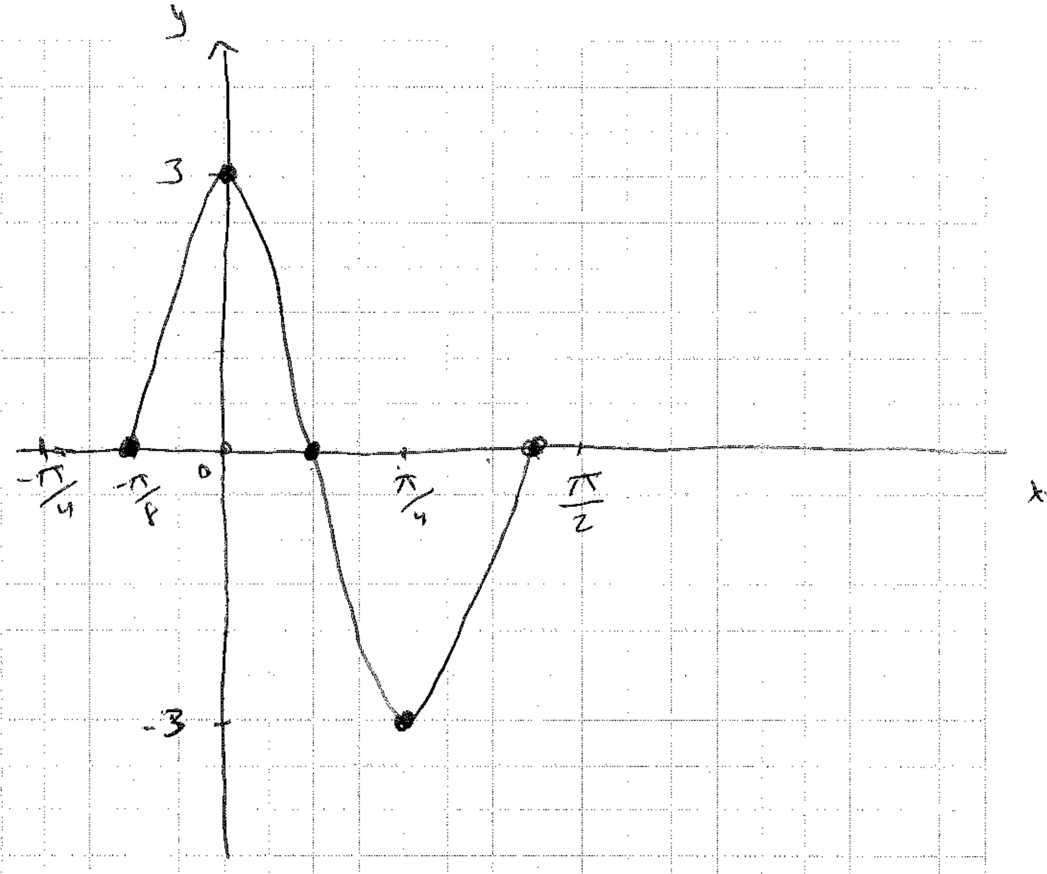
$y = 3\sin 4(x + \pi/8)$.

Period $= \frac{2\pi}{4} = \pi/2$.

For each x, divide by $\cancel{x}\,4$ and subtract $\pi/8$.
For each y, multiply by 3.

x	y	New x	New y
0	0	$0/4 - \pi/8$	0×3
$\pi/2$	1	$\pi/2 \div 4 - \pi/8$	1×3
π	0	$\pi/4 - \pi/8$	0×3
$3\pi/2$	-1	$3\pi/2 \div 4 - \pi/8$	-1×3
2π	0	$\frac{2\pi}{4} - \frac{\pi}{8}$	0×3

New x	New y
$-\pi/8$	0
0	3
$\pi/8$	0
$\pi/4$	-3
$3\pi/8$	0



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Graph of Sec x

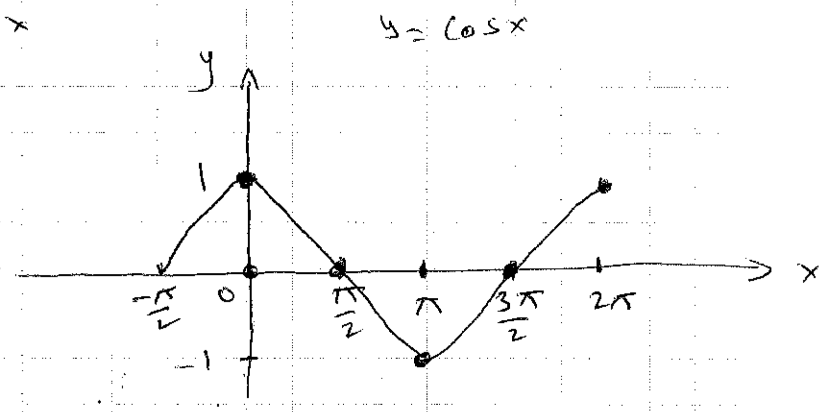
Example 1: Graph $y = \sec x$

\- Solution -

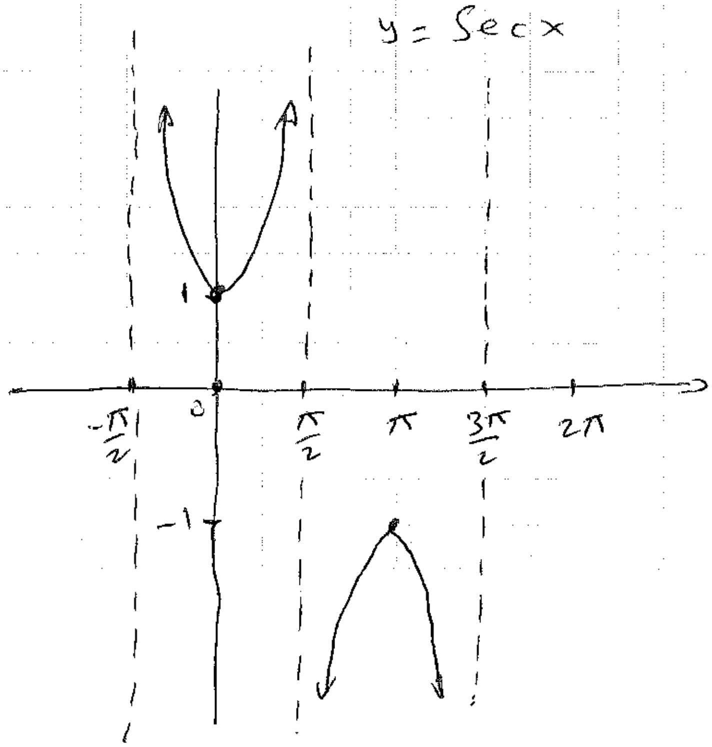
$\sec x = \frac{1}{\cos x}$. Domain is for all x except when the denominator $= 0$. $\cos x = 0$ when $x = \pi/2$ or $x = \frac{3\pi}{2}$. Therefore $x = \pi/2$ and $x = 3\pi/2$ are the vertical asymptotes

Now graph $y = \cos x$

x	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1



When the graph opens upward on cos x, it opens downward on Sec x



Example 2:

Graph. $y = \sec(x - \pi)$

— Solution —

$\sec x = \dfrac{1}{\cos x}$; $y = \dfrac{1}{\cos(x-\pi)}$

Graph: $y = \cos(x - \pi)$

Vertical asymptotes occur when the denominator $= 0$.

$\cos(x - \pi) = 0$, but $\cos \frac{\pi}{2} = 0$.

$\cos(x - \pi) = \cos \frac{\pi}{2}$

$x - \pi = \frac{\pi}{2}$ or $\cancel{x} = \frac{3\pi}{2}$ ✓

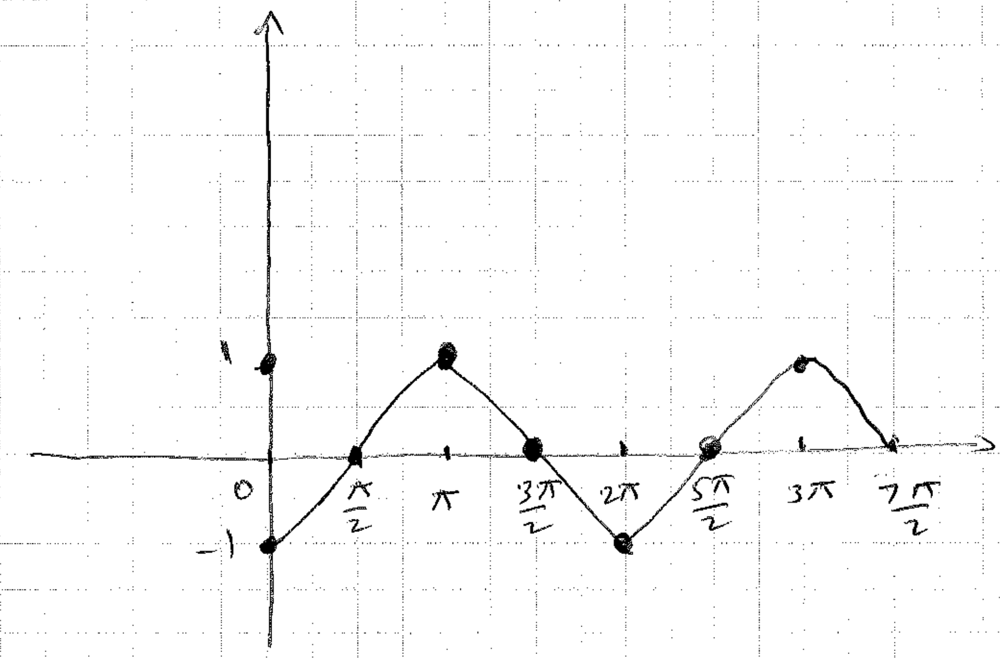
Also $\cos \frac{3\pi}{2} = 0$

$\cos(x - \pi) = \cos \frac{3\pi}{2}$

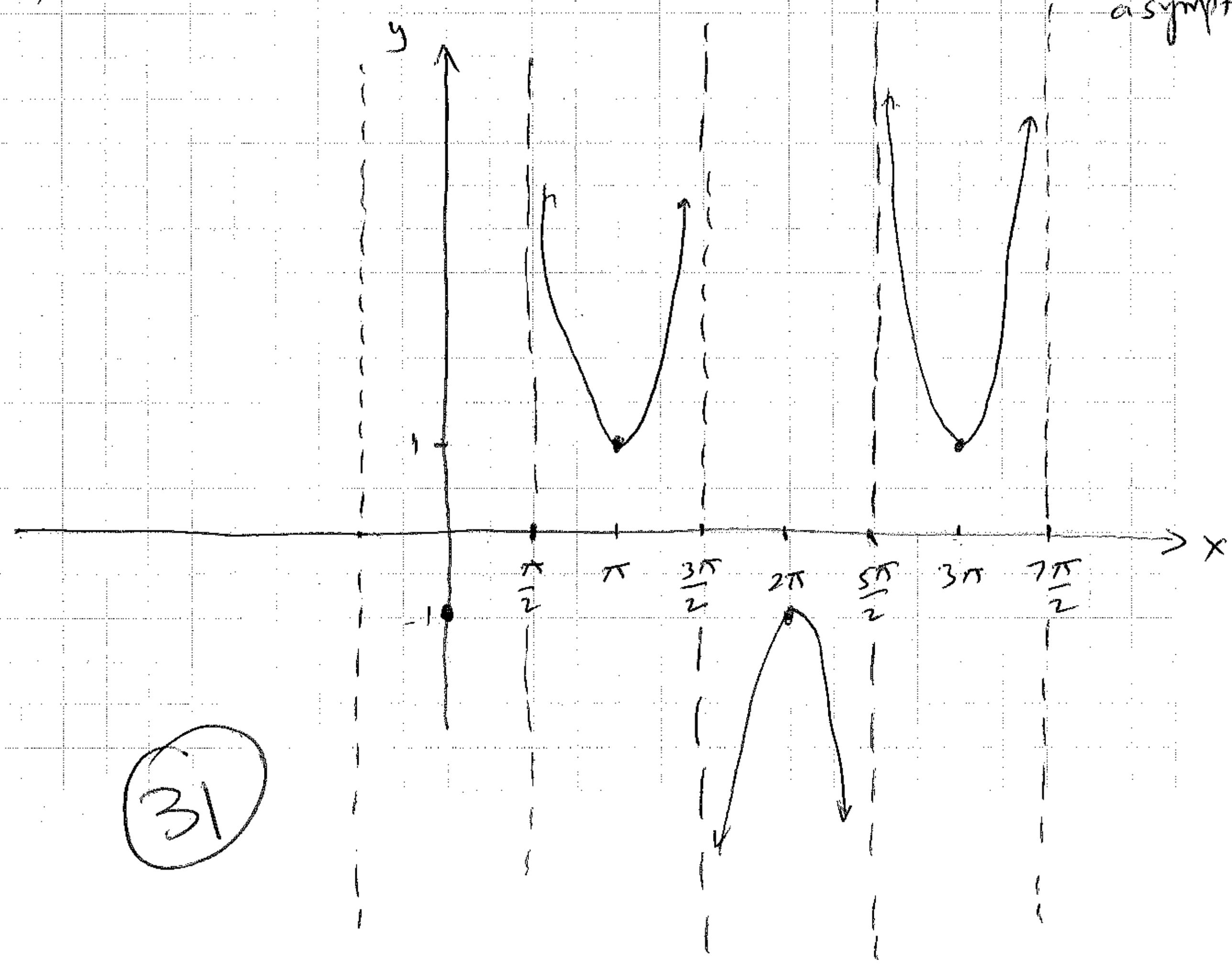
$x - \pi = \frac{3\pi}{2}$, $x = \frac{5}{2}\pi$ ✓

x	new x	New x	y
0	$0 + \pi$	π	1
$\frac{\pi}{2}$	$\pi/2 + \pi$	$3\pi/2$	0
π	$\pi + \pi$	2π	-1
$\frac{3\pi}{2}$	$\frac{3\pi}{2} + \pi$	$\frac{5\pi}{2}$	0
2π	$2\pi + \pi$	3π	1

Graph of $y = \cos(x - \pi)$



Graph of $y = \sec(x - \pi)$. x-intercepts of $\cos(x-\pi)$ are the asymptotes.



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Example 1: Graph : $y = \csc x$

– Solution –

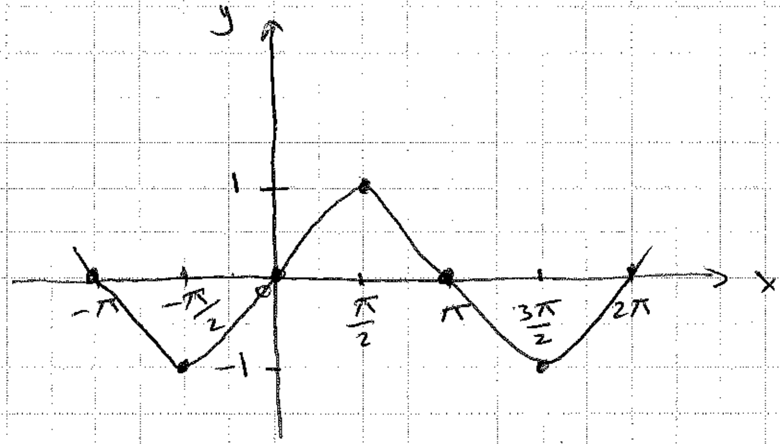
$\csc x = \dfrac{1}{\sin x} \rightarrow$ Vertical asymptotes are when

$\sin x = 0$, $x = 0$, $x = \pi$, $x = 2\pi$.

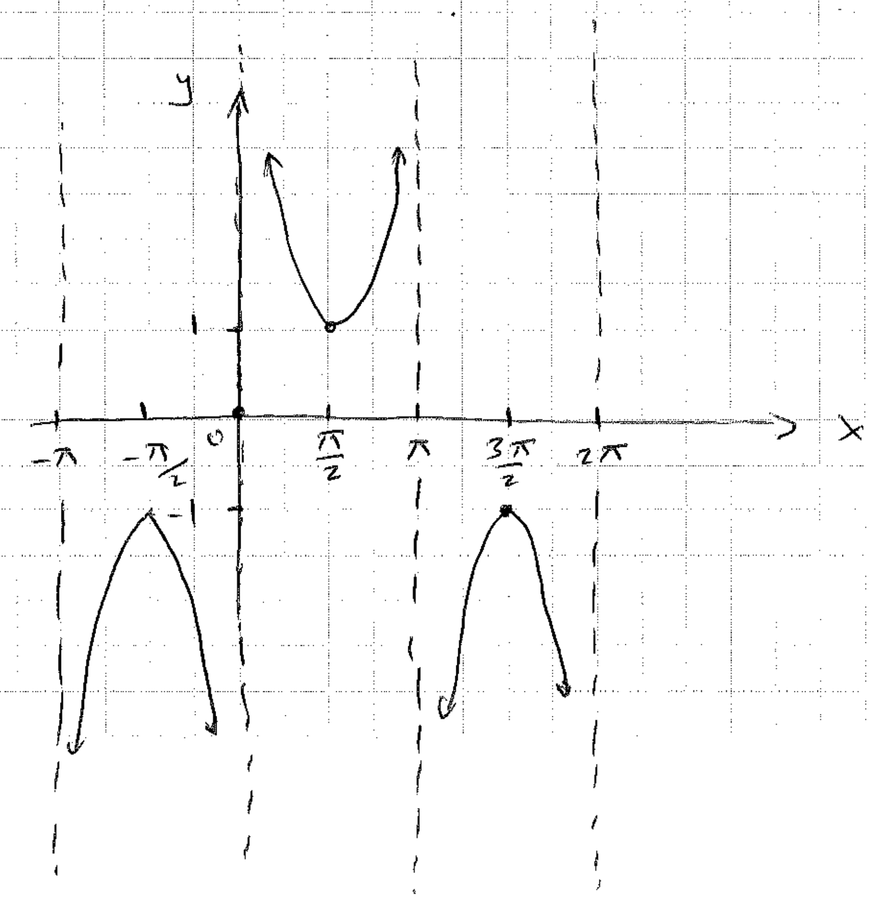
Also at $x = -\pi$, $x = -2\pi$ and so on.

first graph $y = \sin x$

x	y
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0
$-\pi/2$	-1
$-\pi$	0



$y = \csc x$ graph



Example 1:

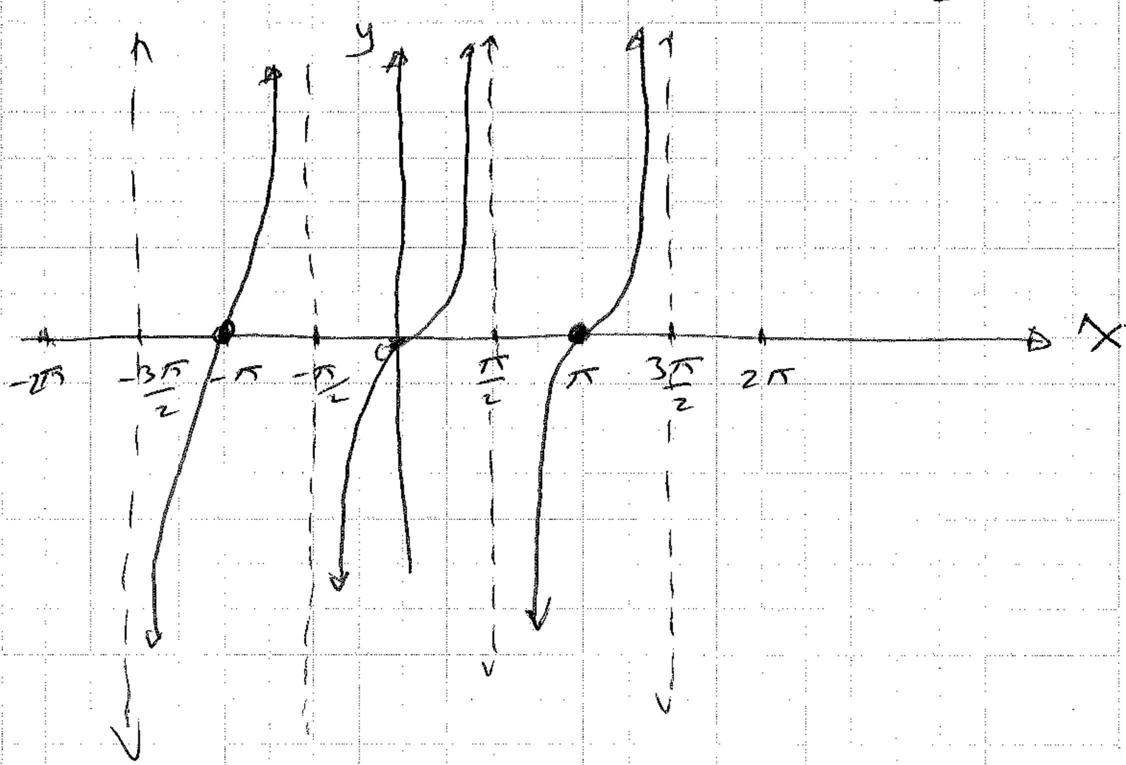
Graph: $y = \tan x$

– Solution –

$$y = \tan x = \frac{\sin x}{\cos x}$$

The vertical asymptotes are when the denominator $= 0$.

$\cos x = 0 \Rightarrow x = \frac{\pi}{2}$, $x = -\frac{\pi}{2}$, $x = \frac{3\pi}{2}$



Example 2: Graph: $y = 3\tan\left(2x - \frac{\pi}{2}\right)$

– Solution –

$y = 3\tan(2x - \pi/2)$. Take 2 as a common factor.

$y = 3\tan 2(x - \pi/4)$

The vertical asymptotes are when $\tan x$ is undefined at $x = \pi/2$ and $x = 3\pi/2$.

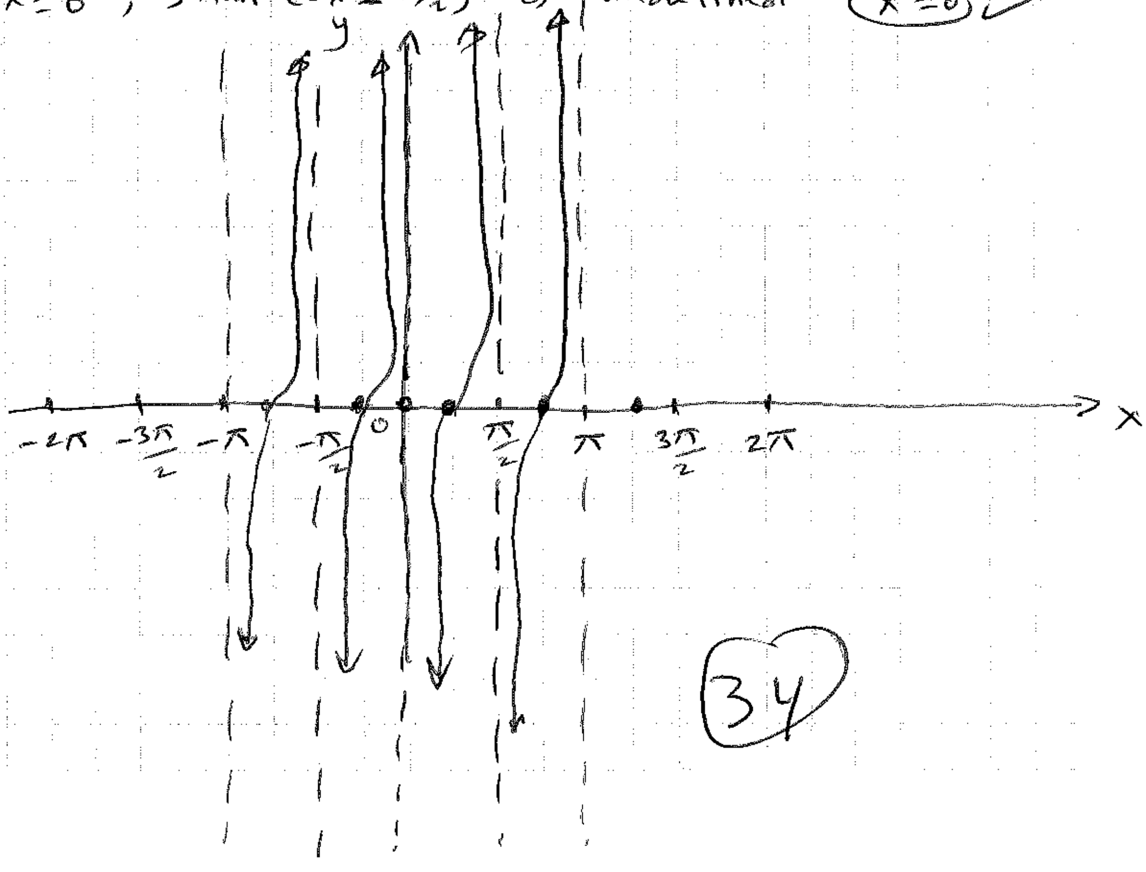
$\tan\left(2x - \frac{\pi}{2}\right) = \tan\frac{\pi}{2}$

$\Rightarrow 2x - \pi/2 = \pi/2$ or $2x = \pi$ or $x = \pi/2$

or $\tan\left(2x - \frac{\pi}{2}\right) = \tan\frac{3\pi}{2}$

$2x - \pi/2 = \frac{3\pi}{2}$ or $2x = 2\pi$ or $x = \pi$

Also at $x = 0$, $3\tan(2x - \pi/2)$ is undefined $x = 0$



Find the x-intercepts: Let $y = 0$

$$0 = 3\tan\left(2x - \frac{\pi}{2}\right)$$

$$\tan 0 = \tan\left(2x - \frac{\pi}{2}\right)$$

$$0 = 2x - \pi/2 \quad ; \quad 2x = \pi/2 \Rightarrow x = \pi/4$$

$(\pi/4, 0)$ ✓

Also $\tan \pi = 0$.

$$\tan \pi = 3\tan\left(2x - \frac{\pi}{2}\right)$$

$$\pi = 2x - \pi/2 \quad \text{or} \quad 2x = 3\pi/2 \quad \text{or} \quad x = 3\pi/4.$$

$(3\pi/4, 0)$. ✓

Example 3: Graph $y = \tan 2x$

– Solution –

$$\tan 2x = \frac{\sin 2x}{\cos 2x}$$

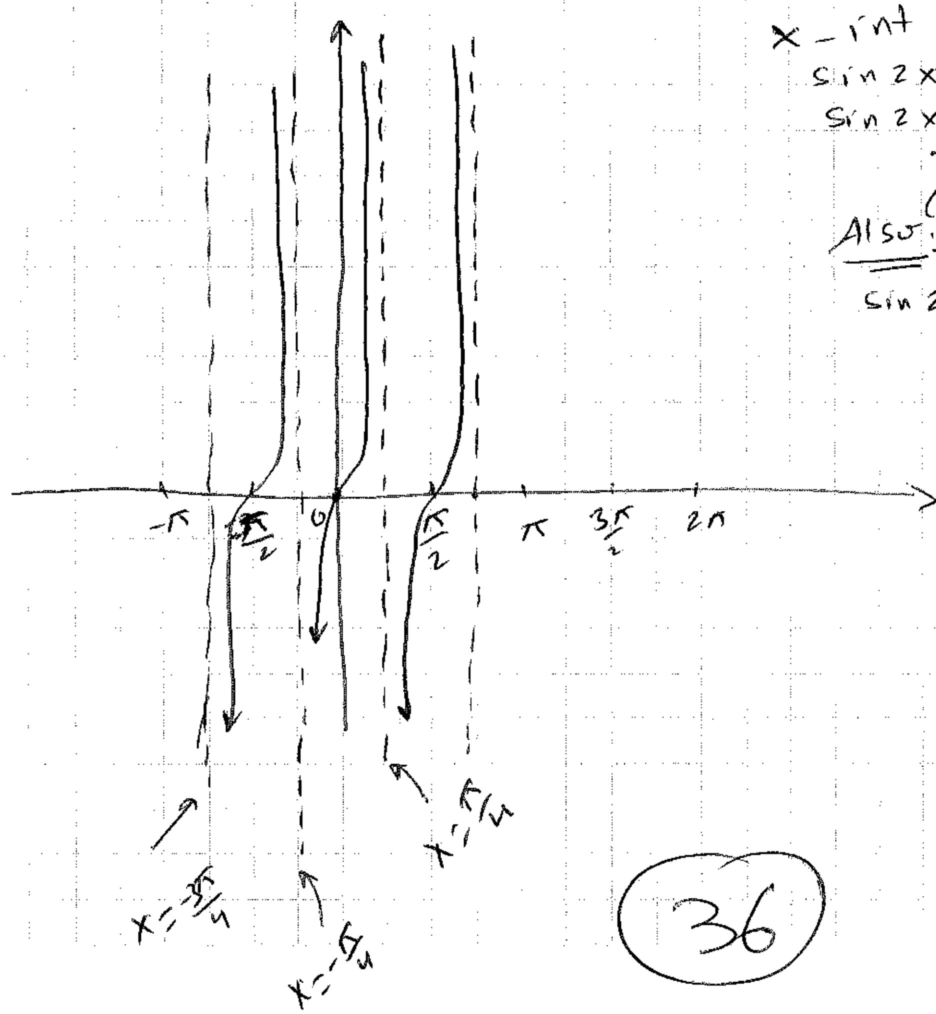
The vertical asymptotes are:

$\cos 2x = 0 \Rightarrow \cos 2x = \cos\frac{\pi}{2}$

$\Rightarrow 2x = \pi/2 \Rightarrow x = \pi/4.$

Also $\cos 2x = 0 \Rightarrow \cos 2x = \cos\frac{3\pi}{2}$

$\Rightarrow 2x = 3\pi/2 \Rightarrow x = 3\pi/4$



x-int $\Rightarrow y = 0$

$\sin 2x = 0$

$\sin 2x = \sin 0$

$\Rightarrow x = 0$

$(0, 0)$

Also:

$\sin 2x = \sin\pi$

$2x = \pi$

$x = \pi/2$

$(\pi/2, 0)$

Example 1:

Graph: $y = \cot x$

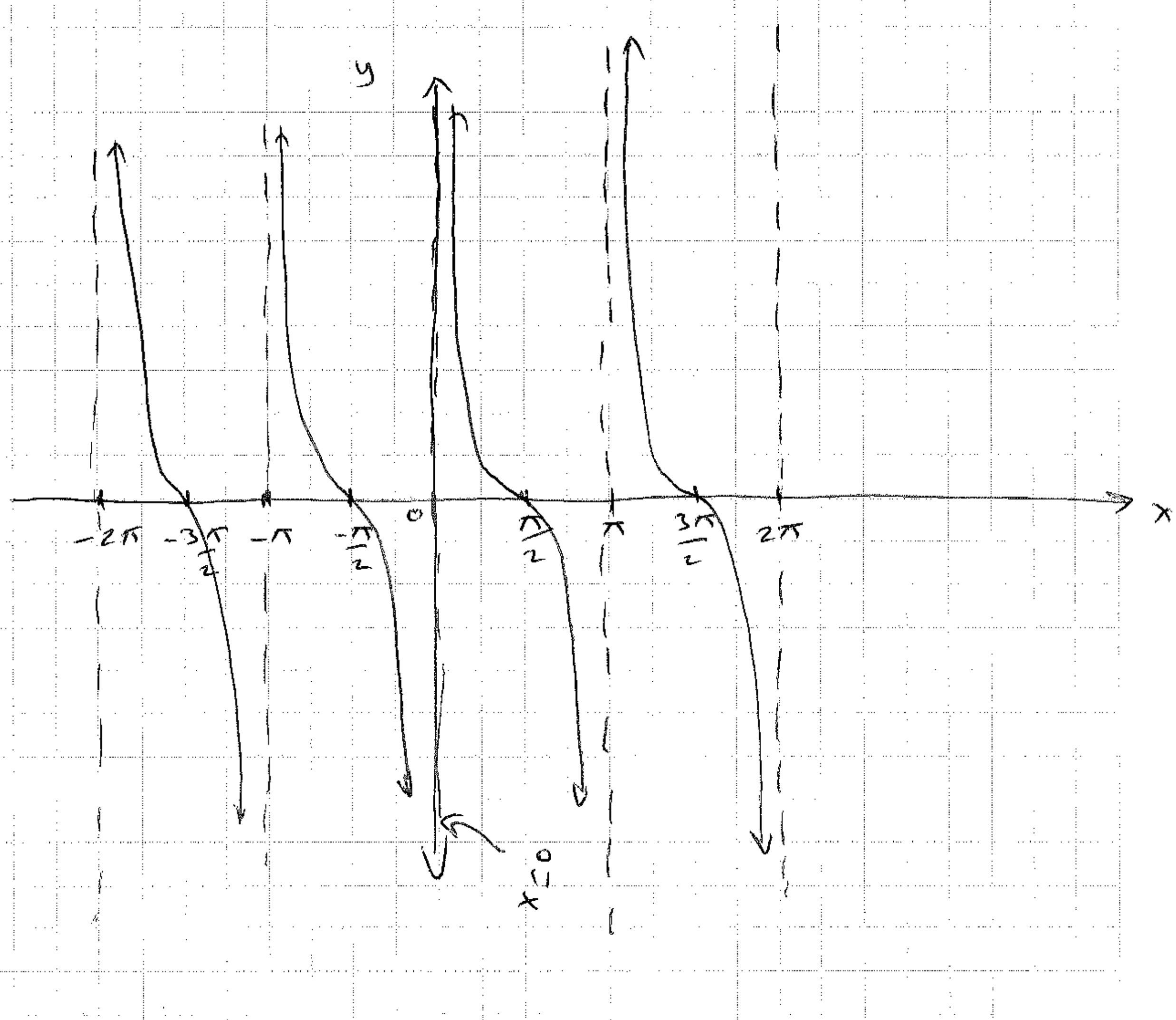
— Solution —

$$y = \cot x = \frac{\cos x}{\sin x}$$

Vertical asymptotes are when $\sin x = 0$.

$\sin x = 0$ when $x = 0$, $x = \pi$, $x = 2\pi$

or $x = -\pi$, $x = -2\pi$



Example 2: Graph: $y = \cot \frac{1}{2} x$

- Solution -

$$\cot \tfrac{1}{2} x = \frac{\cos \frac{1}{2} x}{\sin \frac{1}{2} x}$$

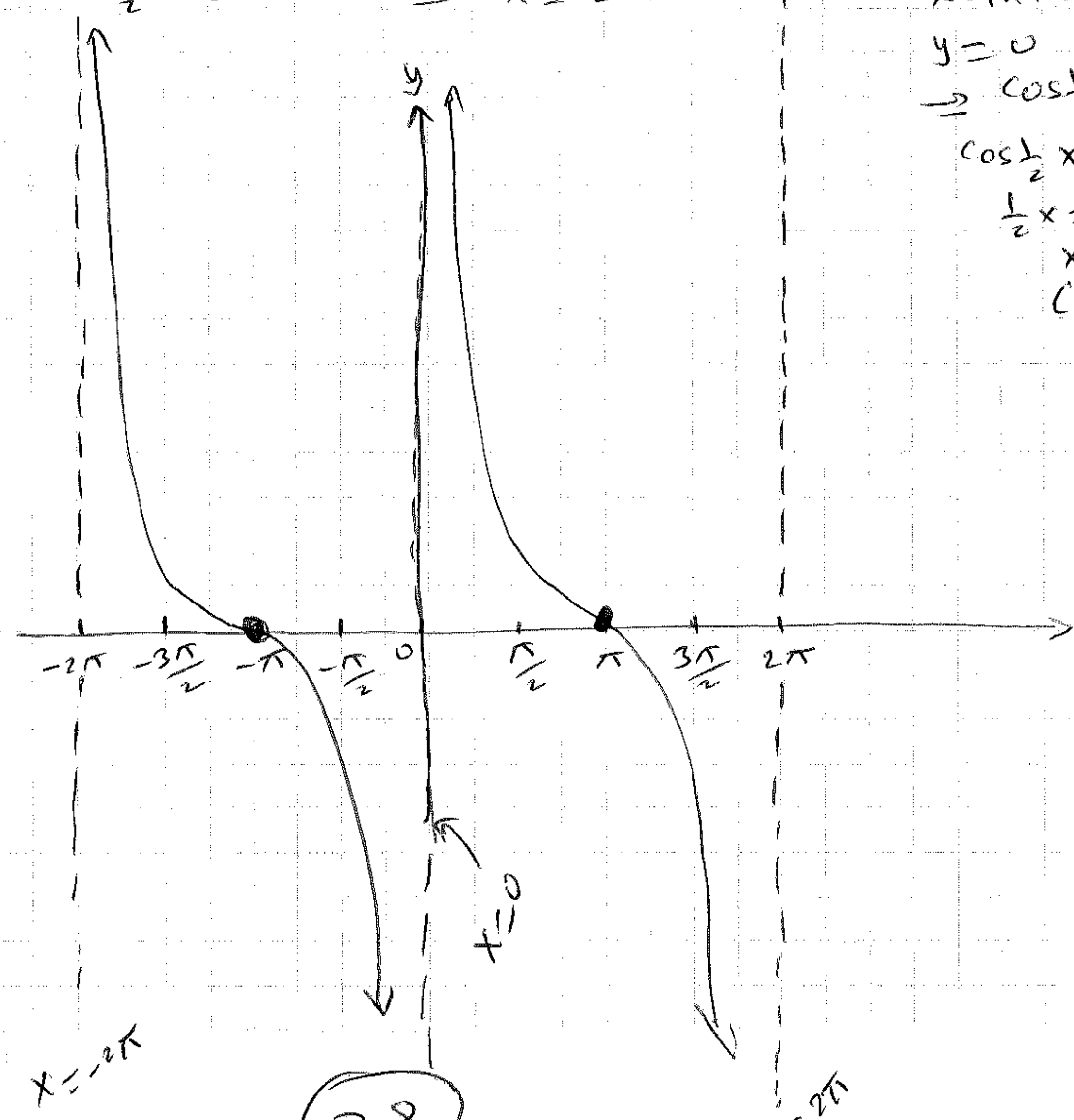
Vertical asymptotes are when $\sin \frac{1}{2} x = 0$

$\sin \frac{1}{2} x = \sin 0 \Rightarrow \frac{1}{2} x = 0 \Rightarrow x = 0.$

$\sin \frac{1}{2} x = \sin \pi$ (because $\sin \pi = 0$)

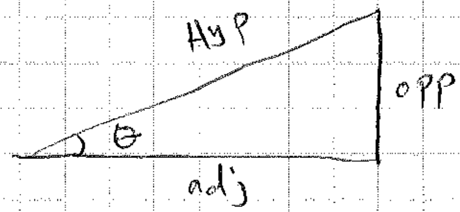
$\frac{1}{2} x = \pi \Rightarrow x = 2\pi$

x-int $\Rightarrow$
$y = 0$
$\Rightarrow \cos \frac{1}{2} x = 0$
$\cos \frac{1}{2} x = \cos \frac{\pi}{2}$
$\frac{1}{2} x = \frac{\pi}{2}$
$x = \pi$
$(\pi, 0)$



Given 1 Trig function, find the
~ remaining 5 Trig functions ~

Background information.



$\sin\theta = \dfrac{opp}{Hyp}$

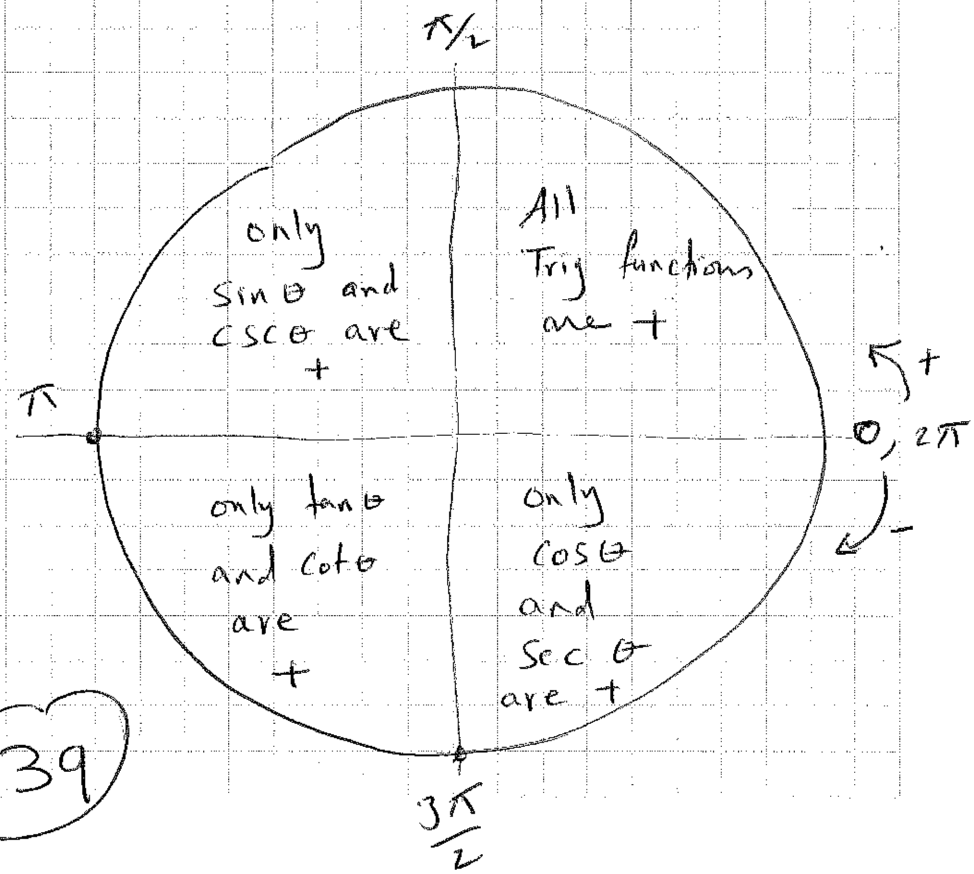
$\csc\theta = \dfrac{Hyp}{opp}$

$\cos\theta = \dfrac{adj}{Hyp}$

$\sec\theta = \dfrac{Hyp}{adj}$

$\tan\theta = \dfrac{opp}{adj}$

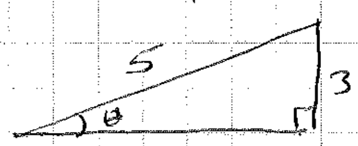
$\cot\theta = \dfrac{adj}{opp}$



Example 1: θ is in quadrant I and $\sin\theta = \frac{3}{5}$, find the remaining trig functions.

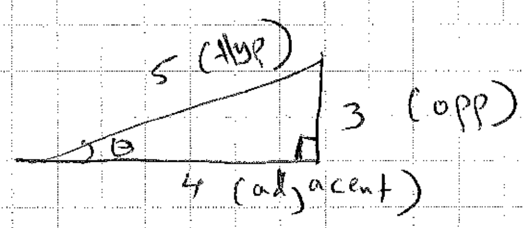
— Solution —

$$\sin\theta = \frac{3}{5} = \frac{opp}{Hyp} \Rightarrow opp = 3 \text{ and } Hyp = 5.$$



Using Pythagorean theorem, the adjacent side =

$$\sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$$



$$\csc\theta = \frac{Hyp}{opp} = \frac{5}{3}$$

$$\cos\theta = \frac{adj}{Hyp} = \frac{4}{5}$$

$$\sec\theta = \frac{Hyp}{adj} = \frac{5}{4}$$

$$\tan\theta = \frac{opp}{adj} = \frac{3}{4}$$

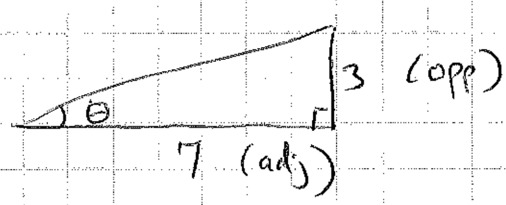
$$\cot\theta = \frac{adj}{opp} = 4/3$$

Since θ is in Quadrant I, they are all +

Example 2: θ is in quadrant II and $\tan\theta = \frac{-3}{7}$, find the remaining trig functions.

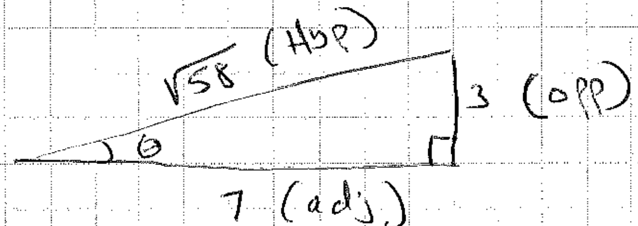
— Solution —

$$\tan\theta = \frac{-3}{7} = \frac{opp}{adj} \Rightarrow opp = 3 \text{ and } adj = 7.$$



Using Pythogorean Theorem, the hypotenus =

$$\sqrt{3^2+7^2} = \sqrt{9+49} = \sqrt{58}.$$



$\cot\theta = \frac{adj}{opp} = \frac{7}{3}$ (since θ is in quadrant II, it is (−)

$\cot\theta = -7/3$ ✓

$\sin\theta = \frac{opp}{Hyp} = \frac{3}{\sqrt{58}} = \frac{3}{\sqrt{58}} \times \frac{\sqrt{58}}{\sqrt{58}} = \frac{3\sqrt{58}}{58}$ ✓

(Since θ is in quadrant II, it is (+).

$\csc\theta = \frac{Hyp}{opp} = \frac{\sqrt{58}}{3}$ (it is (+).) ✓

$\cos\theta = \frac{adj}{Hyp} = \frac{7}{\sqrt{58}} = \frac{7}{\sqrt{58}} \times \frac{\sqrt{58}}{\sqrt{58}} = \frac{7\sqrt{58}}{58}$ and

Since θ is in quadrant II, it is (−)

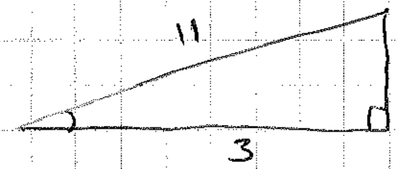
$\cos\theta = \frac{-7\sqrt{58}}{58}$ ✓

$\sec\theta = \frac{Hyp}{adj} = \frac{\sqrt{58}}{7}$. (Since θ is in quadrant II $\Rightarrow$ it is (−)) $\Rightarrow \sec\theta = \frac{-\sqrt{58}}{7}$ ✓

Example III: θ is in quadrant III and $\cos\theta = \frac{-3}{11}$. Find the remaining trig functions

– solution –

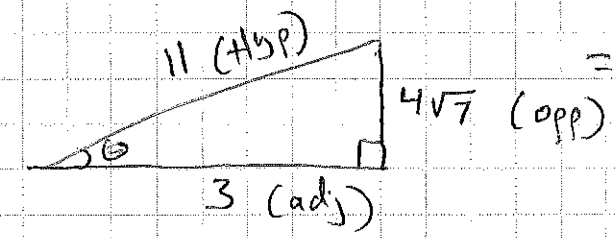
$$\cos\theta = \frac{-3}{11} = \frac{adj}{Hyp} \Rightarrow adj = 3 \text{ and } Hyp = 11$$



Using Pythogorean Theorem, the opposite side =

$$\sqrt{11^2 - 3^2} = \sqrt{121-9} = \sqrt{112}$$

$$= \sqrt{16\times 7} = 4\sqrt{7}$$



$\sec\theta = \frac{Hyp}{adj} = \frac{11}{3}$ since θ is in quadrant III, it is $(-)$

$\sec\theta = \frac{-11}{3}$ ✓

$\sin\theta = \frac{opp}{Hyp} = \frac{4\sqrt{7}}{11}$ (since θ is in quadrant III, it is $(-)$.

$\sin\theta = \frac{-4\sqrt{7}}{11}$ ✓

$$\csc\theta = \frac{Hyp}{opp} = \frac{11}{4\sqrt{7}} = \frac{11}{4\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{11\sqrt{7}}{28}$$

(Since θ is in quadrant III, $\csc\theta$ is $(-)$.

$\csc\theta = \frac{-11\sqrt{7}}{28}$ ✓

$\tan\theta = \frac{opp}{adj} = \frac{4\sqrt{7}}{3}$ ✓ (stays +)

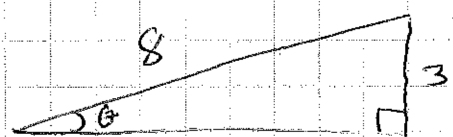
$$\cot\theta = \frac{adj}{opp} = \frac{3}{4\sqrt{7}} = \frac{3}{4\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{3\sqrt{7}}{28}$$ ✓

(stays +)

Example IV: θ is in quadrant IV and $\csc\theta = \frac{-8}{3}$. Find the remaining trig functions

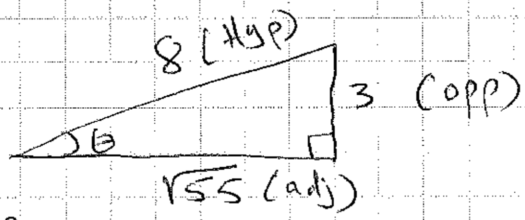
— Solution —

$$\csc\theta = \frac{-8}{3} = \frac{Hyp}{opp} \Rightarrow Hyp = 8 \text{ and } opp = 3$$



Using Pythagorean Theorem, the adjacent side =

$$\sqrt{8^2 - 3^2} = \sqrt{64 - 9} = \sqrt{55}$$



$\sin\theta = \frac{opp}{Hyp} = \frac{3}{8}$ (Since θ is in quadrant IV, $\sin\theta$ is $(-)$).

$\sin\theta = \frac{-3}{8}$ ✓

$\cos\theta = \frac{adj}{Hyp} = \frac{\sqrt{55}}{8}$ ✓ (Since θ is in quadrant IV, $\cos\theta$ is $(+)$.)

$\sec\theta = \frac{Hyp}{adj} = \frac{8}{\sqrt{55}} = \frac{8}{\sqrt{55}} \times \frac{\sqrt{55}}{\sqrt{55}} = \frac{8\sqrt{55}}{55}$ ✓

(Since θ is in quadrant IV, $\sec\theta$ is $(+)$)

$\tan\theta = \frac{opp}{adj} = \frac{3}{\sqrt{55}} = \frac{3\sqrt{55}}{55}$ (Since θ is in quadrant IV, $\tan\theta$ is $(-)$).

$\tan\theta = \frac{-3\sqrt{55}}{55}$ ✓

$\cot\theta = \frac{adj}{opp} = \frac{\sqrt{55}}{3}$ (Since θ is in quadrant IV) $\cot\theta$ is $(-)$).

$\cot\theta = \frac{-\sqrt{55}}{3}$ ✓